REPORT RESUMES

ED 003 241 56

VARIATIONS IN INSTRUCTIONAL MEDIA, PROCESSES, CONTENT AND APTITUDE VARIABLES IN RELATION TO EFFICIENCY OF COGNITIVE GOAL ATTAINMENT, VOLUME 3.

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INDIANA UNIV., BLOOMINGTON, INST. OF EDUC. RES.

REPORT NUMBER NDEA-VIIA-962-VOL-3 PUB DATE 65
GRANT OEG-7-24-0210-185
EDRS PRICE MF-$0.54 HC-$14.68 367P.

DESCRIPTORS- *EDUCATIONAL RESEARCH, *MEDIA RESEARCH, *PROGRAMED INSTRUCTION, *AUDIOVISUAL INSTRUCTION, BLOOMINGTON, INDIANA

THE REPORTS UNDER ACCESSION NUMBERS ED 003 239 AND ED 003 240 ARE SUPPLEMENTED BY THIS DOCUMENT. THREE APPENDIXES ARE INCLUDED. THE FIRST SHOWS INTERCORRELATIONS AND FACTOR LOADINGS FOR 27 MEASURES USED IN A STUDY OF SYMBOLIC LOGIC. THE SECOND PROVIDES THREE PROGRAMED INSTRUCTIONAL BOOKLETS IN THE FIELD OF STATISTICS FOR GRADUATE COLLEGE STUDENTS MAJORING IN EDUCATION AND OTHER FIELDS OF SOCIAL OR APPLIED SCIENCES--(1) "ANALYSIS OF VARIANCE," (2) "MULTIPLE COMPARISONS AMONG TREATMENTS," AND (3) "CORRELATION." THE THIRD DESCRIBES A GENERAL-PURPOSE, PROCEDURE-ORIENTED, COMPUTER PROGRAMING LANGUAGE (P.O.L.). (JH)

ED003241

A-962
Vol III

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Title VII Project Number 962

National Defense Education Act of 1958

Grant Number 7-24-0210-185

N. A. Fattu
Principal Investigator

Institute of Educational Research
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APPENDIX VIII

INTERCORRELATIONS AND FACTOR LOADINGS OF 27 MEASURES USED IN THE SYMBOLIC LOGIC STUDY

TESTS USED IN THE SYMBOLIC LOGIC STUDY

1. Study Guide - Knowledge
2. Study Guide - Comprehension
3. Study Guide - Application
4. Post Test - Knowledge
5. Post Test - Comprehension
6. Post Test -]Application
7. Brown-Carlsen - A
8. Brown-Carlsen - B
9. Watson-Glaser
10. Ship Destination
11. Otis I. Q.
12. Guilford-Zimmerman, Verbal Comprehension
13. Guilford-Zimmerman, General Reasoning
14. Guilford-Zimmerman, Numerical Operations
15. Guilford-Zimmerman, Perceptual Speed
16. Guilford-Zimmerman, Spatial Orientation
17. Guilford-Zimmerman, Spatial Visualization
18. Guilford-Zimmerman, Mechanical Knowledge
19. Christensen-Guilford Fluency Tests: (a) Word Fluency
20. Christensen-Guilford Fluency Tests: (b) Ideational Fluency
21. Christensen-Guilford Fluency Tests: (c) Associational Fluency
22. Christensen-Guilford Fluency Tests: (d) Expressional Fluency
23. Vocabulary Analogies test (locally constructed)
24. SCAT - Verbal
25. SCAT - Quantitative
26. Thurstone Test of Mental Alertness - Quantitative
27. Thurstone Test of Mental Alertness - Language

PRESENTATION #2

FACTOR LOADING

	1	2	3	4	5	6	7	8
1	-.85**	.02	-.05	.08	-.06	-.29	-.04	-.05
2	-.81**	.12	.14	-.05	-.28	.18	-.01	.26
3	-.89**	.01	.10	.02	-.15	.16	-.03	.28
4	-.72**	-.06	-.29	-.09	-.08	-.04	.03	-.12
5	-.87**	-.07	-.02	.01	-.02	.19	-.16	-.09
6	-.36	-.14	.01	-.02	-.65*	-.22	.03	-.29
7	.13	-.50*	-.32	.30	-.31	-.42	.06	-.03
8	-.12	-.28	-.04	.56*	-.58*	.02	-.09	.25
9	-.08	-.02	-.07	-.11	-.89**	-.02	-.15	.12
10	-.30	-.73**	-.12	-.20	-.33	.22	.10	.12
11	-.23	-.75**	-.39	.15	-.19	.05	.23	-.27
12	.16	.03	.78**	.27	-.12	-.15	.43	.15
13	.15	-.38	.25	.24	-.76**	.17	.03	-.23
14	.31	-.08	-.19	.55*	.18	.36	.58*	.02
15	.03	.20	.33	-.12	-.15	-.25	.68*	-.21
16	.15	.06	.73**	-.13	.07	-.23	.19	.06
17	.02	-.13	.02	.92**	.04	-.07	.00	.16
18	.09	-.09	.11	-.07	.21	.13	.89**	.05
19	.24	-.11	.15	-.62*	.08	-.45	.33	.00
20	.04	-.14	.04	-.41	-.02	-.04	.01	-.81**
21	.71**	.13	.05	-.16	-.41	.12	.10	.19
22	.18	-.87**	.03	.16	-.02	-.05	-.13	-.00
23	-.47	-.46	.21	-.11	.02	.04	-.11	-.06
24	-.12	-.07	-.17	.01	-.11	.90**	-.03	-.07
25	.25	-.03	-.86**	.22	.07	-.16	.15	.25
26	.01	-.18	-.59*	-.38	.09	-.06	-.20	.56*
27	-.20	.03	-.08	-.16	-.36	-.58*	-.23	-.24

PRESENTATION #3

FACTOR LOADING

	1	2	3	4	5	6	7	8
1	-.04	.14	-.09	-.04	-.13	.91**	.07	-.18
2	-.06	.04	-.09	.14	.02	.91**	.18	.08
3	.11	.12	-.09	.20	-.09	.78**	-.05	.06
4	.41	.04	-.13	.15	-.38	.45	.41	-.11
5	.20	-.25	-.00	-.02	-.74**	.43	.23	-.08
6	.31	-.17	.09	.63*	-.09	.50	.16	-.34
7	.56*	.16	-.13	.25	-.18	.42	-.33	.18
8	.46	-.17	-.44	.05	-.04	.53*	-.02	-.37
9	.29	-.28	.13	-.08	-.34	.42	-.30	-.43
10	.25	-.13	.12	.24	-.74**	-.09	-.02	-.17
11	-.18	.05	-.15	.04	-.13	.05	-.02	-.92**
12	-.26	.82**	-.24	-.14	.17	.12	-.17	-.10
13	-.22	.50*	.28	-.06	-.63*	.24	-.03	.10
14	-.14	.32	.80**	-.03	-.03	-.19	.10	-.05
15	.17	.21	.39	-.41	.65*	-.05	-.01	.15
16	.01	.81**	.25	-.26	.08	.02	.04	-.12
17	-.01	.16	.18	-.01	-.04	.13	.85**	-.06
18	.05	.75	.46	.05	.20	-.02	.10	-.01
19	.01	.03	-.87**	-.13	.02	.06	-.05	-.20
20	.02	.15	.06	-.58*	.12	-.17	-.42	-.29
21	.78	-.37	-.21	-.06	-.17	-.27	.00	-.10
22	-.13	-.23	.09	.59*	-.02	.37*	.02	-.15
23	-.47	-.29	-.09	-.47	-.10	-.30	-.18	.50*
24	.55*	-.42	.34	.13	.09	.43	.34	.12
25	.07	-.88**	-.03	-.07	.03	-.03	-.15	-.16
26	.09	-.10	-.28	.35	-.20	-.08	.39	-.55
27	-.09	-.01	-.08	-.85**	.17	-.06	.11	.10

PRESENTATION #4

FACTOR LOADING

	1	2	3	4	5	6	7	8
1	-.79**	-.06	-.05	-.19	.23	..27	-.03	.00
2	-.85**	.09	.13	.11	-.09	.08	-.18	-.10
3	-.83**	-.15	-.19	-.04	.07	.00	.16	.20
4	-.24	.55*	.05	.26	-.01	.57*	.13	.09
5	-.42	-.29	-.07	-.03	.29	.20	-.17	-.61*
6	-.44	.13	.16	-.22	.48	.35	.36	-.12
7	-.52*	.27	-.02	.11	.60*	.02	.09	.27
8	.11	-.03	-.17	.05	.78**	.08	-.07	.12
9	-.30	.22	.45	.26	.55*	.36	-.01	-.13
10	-.05	.04	.47	-.33	.61*	.06	.28	-.16
11	-.16	.16	.13	-.04	.83**	-.19	-.25	-.02
12	.23	.18	-.14	.78**	-.07	-.39	.02	.04
13	-.51*	.03	-.17	-.73**	.01	-.02	-.23	-.01
14	.27	.37	.01	-.17	-.13	-.00	.45	-.67*
15	.24	-.25	.78**	.08	.26	-.02	-.16	-.14
16	.33	.11	.04	.27	.05	-.81**	.05	-.15
17	-.19	-.04	.00	.73**	-.06	-.35	.36	-.21
18	.13	.38	.28	.13	-.18	-.28	-.06	-.64*
19	-.36	.22	.13	.64*	.13	.22	-.31	.09
20	.06	-.80**	-.07	-.36	.14	.05	.04	.03
21	-.22	-.71	.25	.08	-.19	.44	.13	.08
22	.04	.07	-.11	-.13	.10	.87**	-.13	-.02
23	.08	-.78**	-.13	.25	-.25	-.24	-.11	.08
24	-.03	.09	.89**	-.00	-.14	-.11	.26	.07
25	-.14	-.35	.08	-.25	.51*	.32	-.02	.54*
26	.04	.14	.13	.18	-.09	-.10	.82**	.01
27	-.08	-.82**	.11	-.10	-.13	-.05	-.21	.17

PRESENTATION TOTAL

FACTOR LOADING

	1	2	3	4	5	6	7	8
1	-.26	.01	-.05	.82**	.11	-.03	-.07	.16
2	-.06	-.10	.02	.88**	.02	-.05	.06	-.04
3	-.15	-.00	-.05	.82**	.06	-.05	.03	-.10
4	-.11	.20	.32	.61*	.07	.03	.11	-.01
5	-.18	.19	-.14	.67*	-.25	-.06	.06	-.08
6	-.51*	.25	.32	.46	-.05	.17	-.08	.02
7	-.72**	.01	.09	.12	.23	-.09	.01	.02
8	-.72**	.04	-.16	.08	.10	-.03	.24	.01
9	-.58*	.16	.10	.29	.10	.27	-.02	-.25
10	-.57*	.14	.15	.13	-.37	.02	-.08	-.23
11	-.77**	-.06	.10	.35	-.14	-.02	-.10	-.01
12	-.01	-.81**	-.06	-.13	.29	-.01	.25	-.08
13	-.37	-.32	-.08	.19	-.35	-.44	-.39	-.23
14	-.09	-.25	.49	-.23	-.56*	.02	.13	-.10
15	.03	-.35	-.16	-.16	-.01	.82**	-.18	-.03
16	.03	-.80**	-.06	-.15	-.00	.13	-.01	.12
17	-.07	-.31	-.02	.07	-.16	-.03	.78**	-.01
18	.11	-.68*	.37	-.05	-.22	.23	-.03	-.12
19	-.07	-.04	.09	.05	.81**	.04	-.03	-.24
20	-.00	.03	-.52*	-.16	-.11	.05	-.45	-.23
21	-.05	.25	-.08	-.17	.16	-.01	.03	-.81**
22	-.52	.34	.16	.02	-.09	-.21	.04	.30
23	.29	-.00	-.72**	.01	-.07	-.15	.06	-.15
24	-.06	.22	.21	.24	-.40	.39	.26	-.37
25	-.21	.74**	-.09	-.10	.08	.08	.05	-.16
26	-.03	.33	.27	.03	.03	-.11	.52*	-.16
27	.04	.08	-.76**	.00	.12	.29	-.12	.13

PRESENTATION #2

INTERCORRELATION

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1.00	.67	.72	.61	.63	.40	.01	.17	.17	.17	.24	-.14	-.07	-.33	-.08	-.13	.03	-.19	-.07	.02	-.65	-.10	.25	-.19	-.12	-.07	.25
2		1.00	.90	.45	.64	.29	-.21	.22	.28	.29	.09	.01	.06	-.33	-.08	-.04	-.08	-.07	-.30	-.18	-.36	-.29	.32	.20	-.27	.00	.07
3			1.00	.50	.74	.26	-.26	.32	.24	.33	.01	-.03	-.05	-.28	-.08	-.13	.04	-.04	-.30	-.23	-.50	-.12	.44	.26	-.24	.05	.16
4				1.00	.74	.39	.13	.13	.05	.32	.25	-.38	-.05	-.16	-.12	-.17	-.12	-.06	-.21	.14	-.31	-.14	.38	.03	-.02	.20	.19
5					1.00	.36	-.10	.20	.03	.34	.23	-.24	-.10	-.27	-.18	-.17	-.05	-.25	-.28	-.01	-.55	-.09	.44	.34	-.26	-.04	.12
6						1.00	.32	.26	.56	.43	.31	-.01	.49	-.26	.10	-.01	-.07	-.13	-.06	.22	-.12	.13	.07	-.14	-.20	-.18	.37
7							1.00	.36	.23	.29	.58	-.01	.40	.17	-.12	-.17	.23	-.10	-.00	-.12	.12	.36	.24	-.27	.45	.07	.25
8								1.00	.47	.29	.33	.17	.58	.16	-.14	-.18	.58	-.18	-.42	-.28	.17	.41	.10	.16	.22	.00	.21
9									1.00	.36	.18	-.05	.57	-.31	.05	-.21	-.06	-.37	.01	-.07	.20	.01	.08	.11	-.02	.08	.31
10										1.00	.69	-.16	.41	-.05	-.19	-.16	-.08	.06	-.00	.03	-.17	.54	.28	.32	-.02	.28	-.02
11											1.00	-.28	.51	.22	-.06	-.35	.18	.15	-.06	.28	-.36	.55	.34	.24	.36	.10	.08
12												1.00	.30	.20	.59	.56	.26	.40	.23	-.21	.21	.06	.02	-.24	-.42	-.62	-.13
13													1.00	.12	.05	.13	.17	-.08	-.17	.16	.36	.34	.10	.16	-.23	-.37	.06
14														1.00	.08	-.07	.49	.58	-.25	-.25	.17	.08	-.22	.28	.36	-.17	-.57
15															1.00	.30	-.09	.49	.46	.18	.02	-.23	-.04	-.17	-.22	-.42	.21
16																1.00	-.06	.28	.29	.09	.23	-.14	-.05	-.44	-.56	-.28	-.03
17																	1.00	-.06	-.50	-.43	-.17	.29	-.10	-.06	.24	-.19	-.15
18																		1.00	.24	.07	.18	-.01	-.03	.00	.02	-.13	-.31
19																			1.00	.29	.18	.07	-.08	-.40	-.04	.05	.12
20																				1.00	.03	.11	.12	-.03	-.27	-.25	.31
21																					1.00	-.02	-.28	.00	.16	.07	-.08
22																						1.00	.16	.01	.07	.09	.01
23																							1.00	.15	-.36	-.01	.23
24																								1.00	-.01	-.02	-.29
25																									1.00	.52	.02
26																										1.00	.06
27																											1.00

PRESENTATION #3

INTERCORRELATION

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1.00	.82	.78	.41	.46	.46	.36	.54	.38	.02	.30	.21	.36	-.16	-.12	.13	.20	.07	.19	-.17	-.24	.46	-.32	.27	-.11	.12	-.05
2		1.00	.75	.52	.38	.52	.27	.44	.25	-.06	.00	.13	.17	-.19	-.15	-.32	.25	-.01	.15	-.30	-.30	.49	-.36	.41	-.12	.07	-.21
3			1.00	.48	.44	.46	.45	.35	.23	.06	.10	.14	.16	-.11	-.09	.01	-.08	.10	.06	-.36	-.02	.38	-.34	.25	-.05	.09	-.22
4				1.00	.65	.53	.39	.44	.26	.34	.17	-.15	.15	-.12	-.35	-.02	.28	-.20	.08	-.30	.17	.28	-.59	.49	-.11	.32	-.13
5					1.00	.43	.26	.40	.51	.58	.09	-.30	.28	-.15	-.54	-.14	.19	-.29	-.08	-.27	.26	.29	-.12	.38	.23	.36	-.15
6						1.00	.33	.55	.51	.37	.25	-.26	-.01	-.16	-.29	-.23	.17	-.04	-.07	-.47	.10	.70	-.68	.60	.06	.50	-.63
7							1.00	.53	.26	.15	-.15	-.04	.18	-.18	-.23	.04	-.16	-.00	.06	-.14	.30	.38	-.40	.26	-.04	-.07	-.21
8								1.00	.54	.13	.26	-.06	-.16	-.61	-.29	-.21	.06	-.35	.41	.02	.30	.41	-.46	.33	.21	.42	-.11
9									1.00	.31	.33	-.18	.17	-.20	-.22	-.25	-.12	-.18	.02	.12	.24	.26	-.21	.35	.22	.16	-.21
10										1.00	.18	-.32	.29	.03	-.46	-.13	-.01	-.18	-.09	-.20	.34	.13	-.24	.14	.21	.21	-.37
11											1.00	.13	.01	.11	-.34	.08	-.02	-.05	.42	.10	-.17	.15	-.47	-.14	.17	.50	-.06
12												1.00	.27	.03	.23	.70	-.06	.49	.27	.26	-.54	-.20	-.08	-.60	-.67	-.12	.13
13													1.00	.31	-.17	.32	.25	.35	-.28	.04	-.37	.06	.03	-.16	-.60	-.11	-.09
14														1.00	.31	.51	.19	.59	-.65	-.04	-.30	-.16	-.09	-.04	-.23	-.28	.03
15															1.00	.40	.01	.42	-.26	.28	-.07	-.40	.07	.11	-.16	-.40	.42
16																1.00	.17	.70	-.13	.25	-.39	-.21	-.20	-.28	-.58	-.20	.25
17																	1.00	.31	-.15	-.25	-.16	.15	-.28	.32	-.27	.23	.02
18																		1.00	-.33	.04	-.26	-.11	-.28	-.10	-.65	-.14	-.03
19																			1.00	.02	.18	-.15	-.01	-.28	.09	.08	.14
20																				1.00	-.12	-.40	.14	-.29	-.07	-.21	.34
21																					1.00	-.29	-.03	.35	.41	.19	.02
22																						1.00	-.40	.35	.16	.26	-.41
23																							1.00	-.39	.18	-.33	.48
24																								1.00	.25	.03	-.19
25																									1.00	.03	.08
26																										1.00	-.31
27																											1.00

PRESENTATION #4

INTERCORRELATION

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1.00	.63	.66	.30	.44	.56	.53	.23	.33	.24	.15	-.52	.55	-.28	-.13	-.45	-.14	-.17	.30	.10	.25	.26	-.13	-.10	.38	-.11	.10
2		1.00	.63	.28	.41	.32	.39	-.20	.35	.01	.19	-.08	.40	-.15	-.09	-.38	.19	-.05	.43	-.22	.23	.01	-.12	.09	.04	-.16	-.63
3			1.00	.16	.26	.47	.40	-.02	.14	.04	.09	-.21	.31	-.38	-.28	-.36	.15	-.37	.13	.11	.15	-.05	-.03	-.15	.26	-.05	.22
4				1.00	-.04	.41	.34	-.01	.49	-.01	-.13	.03	-.14	.07	-.18	-.30	.04	.11	.41	-.37	-.04	.35	-.54	.05	.12	.04	-.51
5					1.00	.29	.15	.20	.33	.09	.31	-.28	.30	.09	.10	-.26	-.04	.03	.11	.16	.22	.19	-.02	-.14	.10	-.12	.22
6						1.00	.48	.22	.55	.60	.33	-.48	.22	.13	.14	-.38	-.08	-.09	.10	.08	.05	.27	-.38	.08	.33	.09	-.28
7							1.00	.42	.55	.31	.58	-.04	.22	-.23	-.11	-.14	.07	-.20	.37	-.24	-.07	.04	-.31	-.05	.42	.08	-.21
8								1.00	.19	.30	.40	-.05	.01	-.18	.16	-.02	-.16	-.19	.26	.08	-.16	.09	-.12	-.21	.46	-.06	-.10
9									1.00	.54	.64	-.02	-.12	-.01	.35	-.30	-.10	.04	.40	-.26	.09	.27	-.38	.26	.23	.02	-.20
10										1.00	.51	-.37	.13	.19	.42	-.13	-.06	.09	-.13	.20	.00	.12	-.38	.35	.28	.08	-.13
11											1.00	-.00	.17	-.20	.16	.22	-.03	-.11	.17	.02	-.28	.09	-.43	.00	.33	-.22	-.13
12												1.00	-.65	.02	-.06	.55	.61	.20	.23	-.43	-.28	-.44	.22	-.06	-.44	.19	[illegible]
13													1.00	-.10	-.36	-.36	-.50	-.16	-.13	.23	.02	.12	-.15	-.13	.24	-.29	.05
14														1.00	.09	.16	.10	.53	-.37	-.28	-.26	-.08	-.37	.10	-.51	.38	-.54
15															1.00	.11	.06	.18	.04	.08	.16	-.15	.11	.53	.17	-.11	.25
16																1.00	.46	.55	-.08	-.13	-.46	-.70	.20	.13	-.45	.23	-.14
17																	1.00	.27	.35	-.19	.01	-.40	.19	.11	-.45	.40	-.10
18																		1.00	.10	-.31	-.32	-.27	-.17	.28	-.66	.06	-.40
19																			1.00	-.39	.06	.18	-.08	.09	.00	-.06	-.19
20																				1.00	.56	.07	.50	-.14	.48	-.29	.57
21																					1.00	.26	.46	.21	.41	.03	.53
22																						1.00	-.35	-.21	.22	-.15	-.04
23																							1.00	-.19	.08	-.18	.53
24																								1.00	-.02	.47	.00
25																									1.00	-.20	.26
26																										1.00	-.21
27																											1.00

PRESENTATION TOTAL

INTERCORRELATION

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1.00	.70	.72	.46	.51	.48	.31	.30	.32	.12	.40	-.15	.26	-.23	-.13	-.10	.05	-.07	.15	-.04	-.22	.23	-.09	.04	.05	.02	.07
2		1.00	.73	.41	.48	.36	.14	.18	.26	.12	.33	.02	.24	-.19	-.12	-.13	.13	-.02	.06	-.21	-.13	.08	-.04	.26	-.08	-.03	-.10
3			1.00	.38	.44	.42	.27	.20	.20	.12	.40	-.04	.15	-.25	-.14	-.12	.04	-.09	-.01	-.16	-.10	.13	-.03	.11	.01	.04	.03
4				1.00	.46	.47	.27	.13	.29	.20	.29	-.21	-.03	-.05	-.25	-.18	.07	-.08	.09	-.21	-.05	.12	-.23	.21	.05	.23	-.15
5					1.00	-.36	.11	.24	.32	.32	.42	-.28	.16	-.08	-.23	-.21	.04	-.17	-.07	-.03	-.04	.12	.09	.19	.09	.09	-.05
6						1.00	.38	.29	.54	.43	.46	-.30	.16	-.06	-.06	-.25	.01	-.10	.01	-.13	.03	.40	-.35	.21	.14	.18	-.18
7							1.00	.42	.37	.24	.59	-.03	.22	-.10	-.14	-.06	.03	-.10	.14	-.18	.08	.27	-.22	-.02	.21	.02	-.04
8								1.00	.33	.27	.42	.07	.18	-.17	-.07	-.08	.10	-.20	.08	-.02	.07	.30	-.18	.08	.22	.09	-.02
9									1.00	.37	.54	-.10	.14	-.14	.03	-.25	-.01	-.15	.14	-.06	.17	.18	-.21	.25	.18	.09	-.05
10										1.00	.58	-.25	.30	.08	-.07	-.13	-.03	-.00	-.07	.01	.04	.27	-.13	.28	.16	.19	-.18
11											1.00	-.05	.40	-.05	-.11	-.02	.02	.03	.03	-.04	-.16	.34	-.24	.20	.11	.04	-.15
12												1.00	-.00	.07	.27	.61	.31	.37	.18	-.14	-.19	-.16	.07	-.28	-.54	-.22	-.09
13													1.00	.10	-.14	.09	-.04	.09	-.20	.14	.05	.16	-.02	-.01	-.23	-.23	-.05
14														1.00	.08	.15	.26	.54	-.36	-.18	-.05	-.04	-.25	.15	-.11	.04	-.44
15															1.00	.33	-.06	.37	.04	.16	-.02	-.27	.04	.11	-.14	-.35	.31
16																1.00	.16	.54	-.05	.07	-.24	-.26	-.04	-.23	-.58	-.16	.10
17																	1.00	.19	-.08	-.24	-.07	.03	-.01	.14	-.16	.17	-.11
18																		1.00	-.04	-.08	-.12	-.12	-.17	.06	-.48	-.08	-.24
19																			1.00	-.04	.17	-.00	-.05	-.18	.08	-.07	.01
20																				1.00	.18	-.14	.20	-.15	.10	-.21	.36
21																					1.00	-.06	.07	.20	.35	.14	.06
22																						1.00	-.19	.08	.13	.09	-.13
23																							1.00	-.14	.02	-.15	.38
24																								1.00	.13	.19	-.20
25																									1.00	.16	.05
26																										1.00	-.17
27																											1.00

APPENDIX IX

STATISTICS INSTRUCTIONAL MATERIALS

ANALYSIS OF VARIANCE
(programmed instruction)

Objectives:

The purpose of the programmed material was to acquaint the student of limited mathematical background with the range of alternative designs available. The emphasis was on practical application and use of these principles rather than upon the rigorous mathematical development that underlies them.

This program is intended for those graduate students majoring in Education and other fields of Social or Applied Sciences who are interested in acquiring the skills necessary in treating their research data with the analysis of variance technique.

Upon completion of this program, the student should be able to:

1. Use the technical terms related to the analysis of variance technique.
2. Perform fundamental algebraic operations involving partitions of sums of squares.
3. Identify the assumptions and underlying principles of the analysis of variance technique.
4. Distinguish multi- factor from single-factor experiments.
5. Lay out the schemes of the completely randomized design, randomized block design, randomized treatment design, latin square design, factorial design, and split-plot design, and perform the following:
    - (a) code the experimental data whenever applicable,
    - (b) partition the total experimental variance into appropriate sources of variation for a given experimental design
    - (c) compute the sum of squares for each source of variation in a given experiment
    - (d) determine the number of degrees of freedom for each source of variation in a given experiment.
    - (e) calculate the mean squares (or variance estimates) for appropriate sources of variation in a given experiment

(f) determine appropriate error term (or terms) to be used for a F (or variance-ratio) test in a given experiment

(g) compute the necessary F-ratio (or ratios) in a given experiment

(h) obtain the critical F values from the Table of F for each F - ratio computed in a given experiment

(i) compare the computed F value with its critical F values and interpret the results by referring to the null hypothesis (or hypotheses) postulated in a given experiment

Description :

This is basically a linear program in which the instructional materials are presented in such a way to enable the student to participate actively in the learning. He can proceed at this own individual pace depending upon his previous training, his present state of knowledge, and his aptitude, his tempo or rate of working, . . . By asking the student to construct his responses throughout the program, and by immediate comparison of constructed responses with those that are correct, the program is capable of giving necessary reinforcement for effective learning.

At the end of each section the highlights of that section are summarized by presenting a series of frames reviewing what has been covered.

Validation:

This program was revised twice on the basis of two tryout studies. The more recent validation (second tryout) was carried out during the 1965 summer session at Indiana University. The program used for this tryout study was presented as an instructional aid to an intermediate level educational statistics class to supplement the lectures. This program contained eight sections, 362 frames, and 729 response items. The breakdown was as follows:

Sections	No. of Frames	No. of Responses
1. Introduction	25	43
2. Completely Randomized Design (Equal n's)	57	107
3. Completely Randomized Design (Unequal n's)	26	55
4. Randomized Block Design	40	83
5. Randomized Treatment Design	36	70
6. Latin Square Design	36	74
7. 2x2 Factorial Design (One Observation Per Cell)	52	104
8. 2x2x2 Factorial Design (n Observations Per Cell)	90	193
Total	362	729

Some of the findings of the second tryout study revealed by an item analysis, error analysis and time study were:

1. The program had a Kuder-Richardson reliability coefficient of 0.966.
2. There were 81 frames in which the response items induced an error rate of over 15%. These frames have been revised to lower the difficulty level and reduce the step size of the frames. The distribution of the error rates on the second trial for all the frames in the program is shown below.
3. It took 12 hours and 37 minutes for an average student to complete the program. The breakdown of the average time required for a student to complete all sections of the program in minutes is shown below.

Error No. of Frames Sections	0-5%	6-10%	11-15%	15%	Total
Introduction	4	4	4	13*	25
CRD (Equal n's)	17	16	11	13	57
CRD (Unequal n's)	20	1	2	3	26
RBD	14	1	13	12	40
RTD	25	1	3	7	36
LSD	16	3	8	9	36
2x2 Factorial	28	1	7	16	52
2x2x2 Factorial	51	26	5	8	90
Total	175	53	53	81	362

*Including 2 branching gates (frames)

Section	Average Time (Min.)
Introduction	54
CRD (Equal n's)	108
CRD (Unequal n's)	50
RBD	73
RTD	69
LSD	76
2x2 Factorial	126
2x2x2 Factorial	201
Total	757

Directions:

This program is composed of a series of ordered frames. In each frame a concept or a fact is defined, presented, and explained, and then relevant questions, ranging from one to eight, are posed. The correct answer (or answers) to the question (or questions) asked in a given frame can be found on the right-hand side of the next frame. The student is instructed to follow the pre-arranged sequence of the program step by step.

Now, go to Frame 1. Read the statement carefully and answer the question by constructing your response in the blank provided in the frame. Compare your response with the correct answer given on the right-hand side of the next frame, Frame 2. If your response was wrong, go back to Frame 1, read the statement and the question again. Be sure that you have responded correctly to the question asked before proceeding to the next frame.

If you have tried a frame three times and still cannot get it, stop and ask for help from the laboratory assistant.

N.B.: Most of the time answers are requested at the end of each frame, but sometimes it is necessary to ask for answers at other times; whenever a blank line appears at any place in the frame, __________, an answer should be given.

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June, 1965

ANALYSIS OF VARIANCE

(Programmed Instruction)

by

Meng-shu Tseng

Institute of Educational Research
Indiana University

TABLE OF CONTENTS

Page

I. Introduction (Frames 1 - 25) 1

II. Single-Factor (Or -Classification) Experiments (Frames 26 - 220) 10

1. Completely Randomized Design With Equal n's (Frames 28 - 82) 10

2. Completely Randomized Design With Unequal n's (Frames 83 - 108) 33

3. Randomized Block Design (Frames 109 - 148) 41

4. Randomized Treatment Design - Replicated Measures On Same Subjects (Frames 149 - 184) 56

5. Latin Square Design (Frames 185 - 220) 76

III. Multi-Factor (Or -Classification) Experiments (Frames 221 - 362) 98

1. 2 x 2 Factorial Design Having One Observation Per Cell - Nonreplicated (Frames 228 - 272) 102

2. 2 x 2 x 2 Factorial Design Having n Observations Per Cell - Replicated (Frames 273 - 362) 129

I. INTRODUCTION

(1) **Variance** is defined as the square of the standard deviation. If we know that the standard deviation of a (population or sampling) distribution is 8, then we also know that its variance is equal to ________________.

---

(2) If, on the other hand, the variance of a distribution is already known to be 121, its standard deviation can be obtained by taking the ________________ of 121. Thus, the standard deviation of the distribution is equal to ________________.

64

---

(3) The standard deviation of a statistical population is defined by

$$\sigma = \sqrt{\frac{\Sigma (X-m)^2}{N}} \quad - - - - - - - \text{(I-1)}$$

where σ = standard deviation of the population distribution
X = raw score
m = mean of the population distribution
N = number of cases in the population distribution

Thus, by definition (given in Frame 1), the population variance can be expressed by

σ^2 = ________________.

square root

11

(4) The researcher is very often concerned with the estimation of the population variance based on sample data. Since a small sample tends to underestimate the variance of the parent population, the following formula gives a better estimate of the population variance:

$$s^2 = \frac{\sum(X-\overline{X})^2}{n-1} \text{ - - - - - - - (I-2)}$$

where s^2 = estimate of the population variance
X = raw score
$\overline{X}$ = mean of the sample
n = number of cases in the sample

Notice that the denominator of the formula is not ________ but n-1.

$\frac{\sum(X-m)^2}{N}$

---

(5) In the analysis of variance, the variance estimate s^2 is more frequently called the mean square, denoted by MS. The variance estimate s^2, or the mean square MS, provides an estimate of the ________ variance.

n

---

(6) The term statistic is used to designate a quantity computed from sample data, and the term parameter is used to designate a quantity characteristic of a population. According to our notation, therefore, m, σ, and σ^2, are ________; $\overline{X}$, s, and s^2 are ________.

population

---

(7) The numerator of the computational formula for the variance estimate, $\sum(X-\overline{X})^2$ is usually referred to as the sum of squares, denoted by SS. Suppose a sample composing six subjects has been drawn from a parent population and, after an experiment, yielded the following data: 7, 10, 6, 8, 7, and 4. The mean of these scores is $\overline{X}$ = ________. Thus, the sum of squares is $\sum(X - \overline{X})^2$ = ________.

parameters

statistics

(8) Based on the above data, we may now compute an estimate of the population variance. To do this, we divide the sum of squares $\Sigma(X-\bar{X})^2$ by ______________. (As we shall discuss later, this value represents the number of degrees of freedom). Thus, we have the variance estimate $s^2 =$ ______________.	$\frac{\Sigma X}{N} = \frac{7+10+6+8+7+4}{6} = \frac{42}{6} = 7$ $(7-7)^2+(10-7)^2+(6-7)^2$ $+(8-7)^2+(7-7)^2+(4-7)^2$ $= 20$
(9) There is another way of computing the sum of squares. Instead of $\Sigma(X-\bar{X})^2$, $\Sigma X^2 - (\Sigma X)^2/n$ is used where $\frac{(\Sigma X)^2}{n}$ is known as the correction term. The merit of this computational formula is that you handle only the raw scores. Thus, formula I-2 (Frame 4) becomes $s^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n-1}$ - - - - - (I-3) Therefore, using this formula, the variance estimate of the above example (Frame 7) can be obtained as follows: $s^2 =$ ______________________	$n-1 = 6-1 = 5$ $\frac{\Sigma(X-\bar{X})^2}{n-1} = \frac{20}{5} = 4$
(10) Now, in order to help you find out whether or not you need some enrichment information concerning the theorems about summations, you are asked to do the following. Select one of the answer alternatives provided below which you consider is correct: If $X_i = i(i-1)$, symbol $\sum_{i=1}^{3} X_i$ represents ( ) A. 8 ( ) B. $X_1+X_2+X_3$ ( ) C. I do not know	$\frac{7^2+10^2+6^2+8^2+7^2+4^2 -}{}$ (cont $\frac{\frac{(7+10+6+8+7+4)^2}{6}}{6-1} =$ $\frac{314-\frac{(42)^2}{6}}{5} = \frac{314-294}{5}$ $= 4$

(11) Try again, this time, on the following question by selecting the answer alternative you consider is correct: Let a denote a constant, b represent another constant, and X_i denote a variable. Then, symbol $\sum_{i=1}^{n}(a+bX_i)$ can be best represented by

( ) A. $\sum_{i=1}^{n}a+\sum_{i=1}^{n}bX_i$

( ) B. $na+b\sum_{i=1}^{n}X_i$

( ) C. $a+b\sum_{i=1}^{n}X_i$

( ) D. $a\sum_{i=1}^{n}1+b\sum_{i=1}^{n}X_i$

A. 8 is correct. [$\sum_{i=1}^{3}X_i = X_1+X_2+X_3 =$ $1(1-1)+2(2-1)+3(3-1)$ $= 8.$] Go on to Frame 11.

B. $X_1+X_2+X_3$ is almost correct. However, since $X_i = i(i-1)$, $X_1+X_2+X_3 =$ $1(1-1)+2(2-1)+3(3-1) = 8$. Go on to Frame 11.

C. You need some information skip Frame 11 and work on Frame 12.

---

(12) The summation notation $\sum_{i=1}^{n}X_i$ represents the sum of the values of the ith array with the lower limit 1 and the upper limit n. That is,

$$\sum_{i=1}^{n}X_i = X_1+X_2+\ldots\ldots+X_n \quad\ldots\ldots (1\text{-}4)$$

Suppose $X_1=7$, $X_2=10$, $X_3=6$, $X_4=8$, $X_5=7$, and $X_6=4$ (as was the case in the example presented in Frame 7), the sum of these values can be expressed by __________ . That is,

$$\begin{aligned}\sum_{i=1}^{6}X_i &= X_1+X_2+X_3+X_4+X_5+X_6\\ &= 7+10+6+8+7+4\\ &= 42\end{aligned}$$

where the lower limit of summation is ________, the upper limit is ________.

A. $\sum_{i=1}^{n}a+\sum_{i=1}^{n}bX_i$ can still be simplified. Go on to Frame 12.

B. You are correct. Skip Frames 12-16, and go to Frame 17.

C. The first term is not a but na. Go on to Frame 12.

D. The first term should be na. Go on to Frame 12.

---

(13) Notice that we often omit the limits of summation. That is, notation $\sum X$ is used, instead of $\sum_{i=1}^{n}X_i$, to mean that the summation is to extend over all values of X_i under discussion. Thus, the sum of 7, 10, 6, 8, 7, and 4 can be denoted as follows:

$7+10+6+8+7+4 =$ __________

$\sum_{i=1}^{6}X_i$

1

6

(14) Suppose we have variables X_i and Y_i. The summation of the sum of the two variables is denoted by $\sum_{i=1}^{n}(X_i+Y_i)$.

Let us expand this as follows:

$$\begin{aligned}\sum_{i=1}^{n}(X_i+Y_i) &= (X_1+Y_1)+(X_2+Y_2)+\;.\;.\;.\;.\;+(X_n+Y_n)\\ &= (\underline{\qquad\qquad})+(\underline{\qquad\qquad})\\ &= \sum_{i=1}^{n}X_i+\sum_{i=1}^{n}Y_i\;.\;.\;.\;.\;.\;(I\text{-}5)\end{aligned}$$

That is, the summation of the sum of two or more variables is the sum of their summations (thorem). Therefore,

$$\Sigma(X^2+Y^2) = \underline{\qquad\qquad} + \underline{\qquad\qquad}.$$

ΣX (or ΣX_i)

---

(15) Consider now the summation $\sum_{i=1}^{n}aX_i$ where a is a constant and X_i is a variable. We may expand it as follows:

$$\begin{aligned}\sum_{i=1}^{n}aX_i &= aX_1+aX_2+\;.\;.\;.\;.\;.\;.\;.\;.\;.\;+aX_n\\ &= \underline{\qquad\qquad\qquad\qquad}\\ &= a\sum_{i=1}^{n}X_i\;.\;.\;.\;.\;.\;.\;.\;.\;.\;(I\text{-}6)\end{aligned}$$

That is, a constant factor can be moved across the summation sign (theorem).

$(X_1+X_2+\;.\;.\;+X_n)\;+$
$(Y_1+Y_2+\;.\;.\;+Y_n)$

$\Sigma X^2+\Sigma Y^2$

---

(16) Let $X_1=a$, $X_2=a$, ... $X_n = a$,
then

$$\begin{aligned}\sum_{i=1}^{n}X_i &= X_1+X_2+\;.\;.\;.\;.\;+X_n\\ &= a+a+\;.\;.\;.\;.\;.\;+a\\ &= \underline{\qquad\qquad\qquad\qquad}\end{aligned}$$

That is, $\sum_{i=1}^{n}a = na\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;(I\text{-}7)$

This means that the summation of a constant is equal to the product of that constant and the number of integers from the lower limit through the upper limit of summation (theorem).

$a(X_1+X_2+\;.\;.\;+X_n)$

(17) Suppose that we have a sample consisting of n cases with a total score of ΣX, where X represents the raw score. Then, the mean of the sample can be expressed by $\bar{X} = \frac{\Sigma X}{n}$. Assume that we subtract from each of the n scores mentioned above a constant, a, what would be the magnitude of the new sample mean, $\bar{X}'$, in relation to the original mean, $\bar{X}$? Well, this can be answered as follows:

$$\bar{X}' = \frac{\Sigma(X-a)}{n}$$
$$= \_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_$$
$$= \bar{X} - a$$

That is, the new mean can be obtained by subtracting the ______________ from the original mean.

na

(18) What will, then, happen to the variance estimate, $s^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$, if we subtract a constant, a, from each of the n scores? Let the new variance estimate be denoted by $\bar{s}^2$. The new (or as is often called "coded") Score is X - a, and the new mean is $\bar{X} - a$ (as shown in the preceding frame). Thus,

$$\bar{s}^2 = \frac{\Sigma\left[(X-a) - (\bar{X}-a)\right]^2}{n-1}$$
$$= \_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_$$
$$= s^2$$

That is, the variance estimate will remain ______________

$\frac{\Sigma X - na}{n}$

$\frac{\Sigma X}{n} - \frac{na}{n}$

constant (a)

(19) As you recall in Frame 9 we said that the sum of squares $\Sigma(X-\bar{X})^2$ could be expressed by $\Sigma X^2 - \frac{(\Sigma X)^2}{n}$ where $\frac{(\Sigma X)^2}{n}$ is referred to as the ______________. The relationship may be proved as follows (notice - $\bar{X}$ is a constant, and $\bar{X} = \frac{\Sigma X}{n}$):

$$\Sigma(X-\bar{X})^2 = \Sigma(X^2 - 2\bar{X}X + \bar{X}^2)$$
$$= \_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_$$
$$= \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$

$\frac{\Sigma(X-a-\bar{X}+a)^2}{n-1}$

$\frac{\Sigma(X-\bar{X})^2}{n-1}$

unchanged (or the same)

(20) Analysis of variance (ANOVA), introduced by R. A. Fisher, is one of the most elegant and powerful techniques in modern statistical method. In comparison to the t-test which is appropriate for testing the significance of difference between two treatments, the analysis of variance which uses F-statistic has been designed to provide an efficient test of the significance of differences between two or more treatments. Thus, in comparing three samples in an experiment we should use the ____________________, whereas in comparing only two samples we may use __________________. It must be noted here that the analysis of variance is based on an assumption of normality of the distribution as well as an assumption of homogenity of certain variances. However, it is known that marked departures from normality of the distribution and extreme differences in variance do not greatly affect the F-test as an effective basis for testing the null hypotheses (to be discussed later) in the analysis of variance (ANOVA).

correction term

$\Sigma X^2 - 2\bar{X}\Sigma X + n\bar{X}^2$

$\Sigma X^2 - 2\frac{\Sigma X}{n}\Sigma X + n(\frac{\Sigma X}{n})^2$

$\Sigma X^2 - 2\frac{(\Sigma X)^2}{n} + \frac{(\Sigma X)^2}{n}$

---

(21) You may recall that based on sample data, the population variance estimate can be computed by the formula s^2 = __________________. It is obvious, from this formula, that the analysis of variance is primarily concerned with the computation of (1) the sum of squares, the numerator of the formula, and (2) the number of degrees of freedom, the denominator of the formula (in this case, n - 1). Usually, the values of the sum of squares, number of degrees of freedom, and variance estimate (or as is often called mean square) computed for each source of variation, together with the desired F-ratios are entered in a table like this:

Table of Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Variance Estimate	F-ratio
Total				

analysis of variance
(or F-test)

either t-test or
analysis of variance

(22) The sources of variation, to be entered in the first column of the Table of Analysis of Variance as shown above, are determined by the nature of the research design itself. The underlying rationale for this is that the total variation in a set of data may be partitioned into components associated with possible source of variation whose relative importance we wish to investigate. Therefore, in order to ____________ the total variation, we must know the nature of the research design with which the experimental data have been obtained.

$\frac{\Sigma(X-\bar{X})^2}{n-1}$ (or $\frac{\Sigma x^2}{n-1}$)

---

(23) Assume that a variable Z is composed of two components X and Y such that $Z = X + Y$, with a total of N cases. Let z, x, and y denote the deviations for Z, X, and Y, respectively. Then,

$$\begin{aligned} z &= Z - \bar{Z} \\ &= Z - \frac{\Sigma Z}{N} \\ &= (X+Y) - \frac{\Sigma(X+Y)}{N} \\ &= \underline{\hspace{6cm}} \\ &= \underline{(\hspace{4cm})} + \underline{(\hspace{4cm})} \\ &= x + y \end{aligned}$$

partition

---

(24) Thus, the variance (notice-not variance estimate) of the variable Z is

$$\begin{aligned} \sigma_z^2 &= \frac{\Sigma z^2}{N} \\ &= \frac{\Sigma(x+y)^2}{N} \\ &= \underline{\hspace{8cm}} \\ &= \frac{\Sigma x^2}{N} + \frac{\Sigma y^2}{N} + 2\frac{\Sigma xy}{N} \\ &= \sigma_x^2 + \sigma_y^2 + 2\frac{\Sigma xy}{N} \end{aligned}$$

By definition the product-moment correlation coefficient is $r = \frac{\Sigma xy}{N\sigma_x\sigma_y}$. Substitute this in the above equation, we have

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y$$

When $r = 0$, that is, when X and Y are __________, $2r\,\sigma_x\sigma_y = 0$, and $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$

This means that the total variance can be partitioned into ____________ components of variances

$X+Y - \frac{\Sigma X + \Sigma Y}{N}$

$(X - \frac{\Sigma X}{N}) + (Y - \frac{\Sigma Y}{N})$

(25) Before getting into computation in an analysis of variance, it is often advisable to code the observed data. You may code the data by addition (or subtraction), or multiplication (or division) by a constant without disturbing the analysis. Thus, we may code the observed data shown in Table 1-a by subtracting a constant A from each item of the data and obtain Table 1-b, or we may code the data by dividing each item by a constant B and obtain Table 1-c. The analyses based on these three tables will yield the ________________ results.

Table 1-a Observed Data

Group 1	Group 2	Group 3
X_1	X_5	X_9
X_2	X_6	X_{10}
X_3	X_7	X_{11}
X_4	X_8	X_{12}

Table 1-b

G 1	G 2	G 3
$X_1 - A$	$X_5 - A$	$X_9 - A$
$X_2 - A$	$X_6 - A$	$X_{10} - A$
$X_3 - A$	$X_7 - A$	$X_{11} - A$
$X_4 - A$	$X_8 - A$	$X_{12} - A$

Table 1-c

G 1	G 2	G 3
X_1/B	X_5/B	X_9/B
X_2/B	X_6/B	X_{10}/B
X_3/B	X_7/B	X_{11}/B
X_4/B	X_8/B	X_{12}/B

$$\frac{\sum(x^2+2xy+y^2)}{N}$$

independent (or uncorrelated)

independent

II. Single-Factor (or-Classification) Experiments

(26) In the single-factor (or-classification) experiment, data are classified on the bais of one single experimental variable (i.e. one independent variable). This variable is very often referred to as a factor. The categories (or groups) classified in accordance with the factor are called treatments. Suppose that a researcher is interested in investigating the relative effectiveness of three different teaching methods A, B, and C. In this case, he has a factor called "teaching method" which is composed of three ________ called A, B, and C. This is, in effect, a ________________ experiment.	same
(27) In other words, the primary objective of the single-factor experiment is to compare the ________ ________ of two or more treatments which formulate the ____________ in question.	treatments single-factor (or single-classification

II. 1. Completely Randomized Design with Equal n's

(28) In the completely randomized design the treatments are randomly assigned to the experimental units or vice versa. In educational and psychological research an experimental unit is usually an individual subject. Suppose that the researcher mentioned in Frame 26 has assigned 30 individual subjects at random to the three treatments A, B, and C evenly (i.e. 10 subjects for each treatment). This is said to be a ________ design with an equal n (or treatment size) of ____________.	effects factor

(29) Assume further that after a certain period of experiment he has observed the results as shown in Table 2-a. He may code the observed data prior to the analysis of variance. Table 20b shows the coded data. To obtain this table, evidently he subtracted from each score in Table 2-a a constant, ________, and then the result was further divided by ________.

completely randomized

10

Table 2-a Observed

Treat. A	Treat. B	Treat. C
40	80	90
60	70	60
40	60	50
70	50	40
50	60	30
50	90	60
60	70	70
20	40	50
30	50	30
60	90	50

Table 2-b Coded

Treat. A	Treat. B	Treat. C
-1	3	4
1	2	1
-1	1	0
2	0	-1
0	1	-2
-2	4	1
1	2	2
-3	-1	0
-2	0	-2
1	4	0

---

(30) Suppose that you have been asked to carry out the analysis of variance (ANOVA) with these data. As we have seen the result will be the ________ whether you use the observed data (Table 2-a) or the coded data (Table 2-b). To save your computational time, you would use the ________ data.

50

10

---

(31) You may now start making your analysis step by step following the procedures shown in the Table of ANOVA in Frame 21. According to that Table, the first step is to ________ the total variation into possible components of ________ sources of variation.

same

coded

(treatments A, B, and C, we may compare the deviations of)

(32) To investigate the relative effectiveness of the three treatment means from the grand mean (known as between-treatment variation) with the deviations of the individual scores from their respective treatment means (known as within-treatment variation). In other words, in the completely randomized design, we can partition the total variation into two components: (1) the ____________ variation and (2) the ____________ variation. Complete this step by constructing your Table of ANOVA as follows:

Table 3-a Source of Variation

Source of variation	SS	df	s^2	F
Between T. Within T.				
Total				

partition

independent

---

(33) The next step is to compute the sums of squares for (1) total variation, (2) between-treatment variation, and (3) within-treatment variation. You are now dealing with the column headed SS in the table of ANOVA shown in the preceding frame. Let us use both tables and computional formulas to determine the ________________ (SS's).

between-treatment (or within-treatment

within-treatment (or between-treatment

---

(34) Let

X = coded score

$\bar{X}_1$ = treatment mean for treatment A

$\bar{X}_2$ = " " " " B

$\bar{X}_3$ = " " " " C

$\bar{X}_t$ = grand mean of all the scores

n = number of cases for each treatment

N = total number of cases

The total sum of squares, designated SS_{total}, is the sum of squares of the deviations of all individual scores from their ____________. That is

$$SS_{total} = \sum(X - \bar{X}_t)^2 \quad \ldots\ldots \text{(II-1)}$$

sum of squares

(35) Thus, we must first compute the grand mean of the coded scores $\overline{\overline{X}}_t$. One way in which this can be done is by summing all the 30 coded scores and then dividing this total score by ____________. Let us do this and show the result, together with all the pertinent data, in the following table. (Of course, this is derived from Table 2-b presented in Frame 29.)

grand mean (or $\overline{\overline{X}}_t$)

Table 2-c X, $\sum X$, & $\overline{\overline{X}}_t$

Treat. A n = 10	Treat. B n = 10	Treat. C n = 10	Total N = 30
-1	3	4	
1	2	1	
-1	1	0	
2	0	-1	
0	1	-2	
-2	4	1	
1	2	2	
-3	-1	0	
-2	0	-2	
1	4	0	
Sum -4	16	3	15
Mean -0.4	1.6	0.3	0.5

As shown in the table, each column is summed, and then the three column sums are totaled. That gives $\sum X$ = ________. This value is then divided by N = ____________. Thus, we have the grand mean $\overline{\overline{X}}_t$ = ____________.

30

15

30

0.5

(36) Now, the deviation of each score from the grand mean can be obtained by subtracting the grand mean $\overline{\overline{X}}_t = 0.5$ from it. For instance, the deviation of the first entry of treatment A in Table 2-c is ____________ = -1.5; the deviation of the last entry of treatment C is ____________ = ____________. All the 30 deviations from the grand mean are shown in the following table.

Table 2-d $X - \overline{\overline{X}}_t$ where $\overline{\overline{X}}_t = 0.5$

Treat A	Treat. B	Treat. C
-1.5	2.5	3.5
0.5	1.5	0.5
-1.5	0.5	-0.5
1.5	-0.5	-1.5
-0.5	0.5	-2.5
-2.5	3.5	0.5
0.5	1.5	1.5
-3.5	-1.5	-0.5
-2.5	-0.5	-2.5
0.5	3.5	-0.5

(37) We have all the 30 deviations from the grand mean. If we have each deviation squared, and sum up all of these squared deviations, we would have the ________ The following table shows you exactly what we said we would do.

Table 2-e $(X-\bar{X}_t)^2$ & $\sum(X-\bar{X}_t)^2$

Treat. A	Treat. B	Treat. C.	Total
2.25	6.25	12.25	
0.25	2.25	0.25	
2.25	0.25	0.25	
2.25	0.25	2.25	
0.25	0.25	6.25	
6.25	12.25	0.25	
0.25	2.25	2.25	
12.25	2.25	0.25	
6.25	0.25	6.25	
0.25	12.25	0.25	
Sum 32.50	38.50	30.50	101.50

Each entry is the square of deviation, $(X-\bar{X}_t)^2$. For instance, the first entry of the left column 2.25 is obtained by ________ X ____________. To obtain the total sum of squares (SS_{total}), $\sum(X-\bar{X}_t)^2$, the three columns are summed, and then the column sums are totaled. Thus, SS_{total} = ____________.

-1 -0.5

0 -0.5 = -0.5

---

(38) Frames 35 through 37 presented the so called "Deviation Method". Another more efficient way of arriving at the same result is to use the computational formula:

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} \text{ - - - - - - - - } (II\text{-}1)$$

where X denotes the coded score and N represents the totoal number of cases. Notice that you do not have to compute the grand mean, $\bar{X}_t$. This method, called "Raw Score Method", is useful to those who have access to a calculating machine. According to this method, in order to compute total sum of squares we must know three quantities: ____________, ________ and N (or total number of cases).

total sum of squares (SS_{total})

(-1.5) x (-1.5)

(39) In order to obtain $\sum X^2$ we ought to have X^2, square of coded score, included in our table. To do this an additional column headed X^2 is created for each treatment as shown below.

Table 2-f X, X^2, $\sum X$, & $\sum X^2$

Treat. A		Treat. B		Treat. C		Total	
X	X^2	X	X^2	X	X^2	X	X^2
-1	1	3	9	4	16		
1	1	2	4	1	1		
-1	1	1	1	0	0		
2	4	0	0	-1	1		
0	0	1	1	-2	4		
-2	4	4	16	1	1		
1	1	2	4	2	4		
-3	9	-1	1	0	0		
-2	4	0	0	-2	4		
1	1	4	16	0	0		
Sum -4	26	16	52	3	31	15	109

From this table we obtain $\sum X$ = ______________,
$\sum X^2$ = __________, and N = ________________
Therefore, SS_{total} = __________ - __________
= ______________

$\sum X^2$ (or sum of squares of all the coded scores)

---

(40) We have just obtained SS_{total} by two different methods: one of which is called the __________ method, utilizing the formula $SS_{total} = \sum(X-X_t)^2$; the other is called the __________ method, employing the computational formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

15, 109, and 30

$109 - \frac{(15)^2}{30}$

$= 101.5$

---

(41) To obtain the between-treatment sum of squares, designated $SS_{between}$, we can use either one of the two methods. Let us, first, determine $SS_{between}$ with the deviation method. The between-treatment sum of squares is n times the sum of deviations of the treatment means from the grand mean. That is,

$$SS_{between} = n[(\bar{X}_1-\bar{X}_t)^2+(\bar{X}_2-\bar{X}_t)^2+. . .+(\bar{X}_k-\bar{X}_t)^2] \quad . . (II\text{-}3)$$

where $\bar{X}_k$ denotes the kth treatment mean. Since we have three treatments in our example, the equation becomes:

$SS_{between}$ = ____________________

deviation

raw score

(42) This relation may be shown graphically as follows: Since the total sum of squares, designated SS_{total}, is partitioned into (1) the between-treatment sum of squares, designated $SS_{between}$, and (2) the within-treatment sum of squares, designated SS_{within}, in order to obtain $SS_{between}$ we should eliminate SS_{within} from ___________. To eliminate within-treatment effect, we simply replace each score by its own treatment mean. First, the three treatment means are computed and, together with the grand mean, recorded in the following table.

Table 2-g X, X, & $\bar{\bar{X}}$

	Treat. A. n = 10	Treat. B n = 10	Treat. C n = 10	Total N = 30
	-1	3	4	
	1	2	1	
	-1	1	0	
	2	0	-1	
	0	1	-2	
	-2	4	1	
	1	2	2	
	-3	-1	0	
	-2	0	-2	
	1	4	0	
Sum	-4	16	3	15
Mean	-0.4	1.6	0.3	0.5

$\bar{X}_1 =$ __________ $= -0.4$

$\bar{X}_2 =$ __________ $= 1.6$

$\bar{X}_3 =$ __________ $= 0.3$

$n[(\bar{X}_1-\bar{X}_t)^2+(\bar{X}_2-\bar{X}_t)^2 + (\bar{X}_3-\bar{X}_t)^2]$

---

(43) To get rid of the within-treatment effect each score is, then, replaced by its own _______ as shown below:

Table 2-h $\bar{X}_j$ *

Treat. A $\bar{X}_1$	Treat. B $\bar{X}_2$	Treat. C $\bar{X}_3$
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3
-0.4	1.6	0.3

*$\bar{X}_j$ represents jth treatment mean

SS_{total}

$\frac{-4}{10}$

$\frac{16}{10}$

$\frac{3}{10}$

(44) Next, the deviations of these scores, shown a above, from the grand mean $\bar{X}_t = 0.5$ can be obtained by subtracting from these scores the ______________. The results are given below.

treatment mean

Table 2-i $(\bar{X}_j - \bar{X}_t)$

Treat. A $(\bar{X}_1-\bar{X}_t)$	Treat. B. $(\bar{X}_2-\bar{X}_t)$	Treat. C $(\bar{X}_3-\bar{X}_t)$
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2
-0.9	1.1	-0.2

---

(45) You can now obtain the value of $SS_{between}$ by squaring these deviations and summing them as follows:

grand mean $(X_t = 0.5)$

Table 2-j $(X_j-X_t)^2$ & $\sum(X_j-X_t)^2$

	Treat. A $(\bar{X}_1-\bar{X}_t)^2$	Treat. B $(\bar{X}_2-\bar{X}_t)^2$	Treat. C $(\bar{X}_3-\bar{X}_t)^2$	Total $\sum(\bar{X}_j-\bar{X}_t)^2$
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
	0.81	1.21	0.04	
Sum	8.10	12.10	0.40	20.6

Thus, $SS_{between}$ = ______________

(46) The method just described above is referred to as the deviation method. Another more efficient method, called the raw score method, for computing the between-treatment sum of squares is provided by

$$SS_{between} = \frac{1}{n}\left[(T_1)^2+(T_2)^2+\ .\ .\ .\ .+(T_k)^2\right] - \frac{(\sum X)^2}{N}\ .\ .\ .\ .\ .\ .\ .\ .\ .\ (II\text{-}4)$$

where n = number of cases in each treatment
N = total number of cases
T_k = kth-treatment sum of scores with k representing the number of treatments in the experiment
X = grand total $(T_1+T_2+\ .\ .\ .\ .\ .+T_k)$

Since we have three treatments in our example

$$SS_{between} = \underline{\qquad\qquad\qquad} - \frac{(\sum X)^2}{N}$$

20.6

---

(47) The three treatment totals and the grand total, together with the 30 coded scores, are shown below. Use Formula II-4 and compute the between sum of squares

$$SS_{between} = \underline{\qquad\qquad\qquad} - \underline{\qquad\qquad\qquad}$$
$$= 20.6$$

Table 2-k Treatment Totals & Grand Total

	Treat. A n = 10 X	Treat. B n = 10 X	Treat. C n = 10 X	Total N = 30 X
	-1	3	4	
	1	2	1	
	-1	1	0	
	2	0	-1	
	0	1	-2	
	-2	4	1	
	1	2	2	
	-3	-1	0	
	-2	0	-2	
	1	4	0	
Sum	-4	16	3	15

In Frames (48), (49), and (50), the derivation of Formula II-4 will be presented. Skip these frames and go to Frame (51) if you are not interested.

$$\frac{1}{n}\left[(T_1)^2+(T_2)^2+(T_3)^2\right]$$

(48) Let X_{ij} = score at ith row and jth column
k = number of treatments
n = number of cases in each treatment
N = total number of cases (nk)

Thus, the symbol X_{23} is used to denote the score (raw or coded) at the ________ row and the ________ column. That is, for Table 2-k (Frame 47) X_{23} represents ____________.
Using the notation, then, the jth treatment mean ($\overline{X}_j$) may be represented by

$$\overline{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n}$$

; the grand mean ($\overline{X}_t$) may be denoted by $\overline{X}_t$ = ____________.

$$\frac{1}{10}\left[(-4)^2+(16)^2+(3)^2\right] - \frac{(15)^2}{30}$$

---

(49) As you refer back to Frame 41, the between-treatment sum of squares was defined as

$$SS_{\text{between}} = n[(\overline{X}_1-\overline{X}_t)^2+(\overline{X}_2-\overline{X}_t)^2+\ .\ .\ +(\overline{X}_k-\overline{X}_t)^2]$$

Thus,

$$SS_{\text{between}} = n\sum_{j=1}^{k}(\overline{X}_j-\overline{X}_t)^2$$

$$= n\sum_{j=1}^{k}(\overline{X}_j^{\,2} - 2\overline{X}_t\overline{X}_j + \overline{X}_t^{\,2})$$

$$= n\sum_{j=1}^{k}(\overline{X}_j)^2 - 2nX_t\sum_{j=1}^{k}(\overline{X}_j) + n\sum_{j=1}^{k}(\overline{X}_t)^2$$

$$= \underline{\qquad\qquad} - \underline{\qquad\qquad} + \underline{\qquad\qquad}$$

$$= \frac{1}{n}\sum_{j=1}^{k}\left[\sum_{i=1}^{n}X_{ij}\right]^2 - \frac{\left[\sum_{j=1}^{k}\sum_{i=1}^{n}X_{ij}\right]^2}{N}$$

second

third

1

$$\frac{\sum_{j=1}^{k}\sum_{i=1}^{n}X_{ij}}{nk} \quad \text{or} \quad \frac{\sum_{j=1}^{k}\sum_{i=1}^{n}X_{ij}}{N}$$

(50) In the above equation, $\sum_{i=1}^{n} X_{ij}$ represents the jth column (or treatment) total ; $\sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij}$ denotes the grand total. Let $T_j = \sum_{i=1}^{n} X_{ij}$ and $\sum X = \sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij}$ (refer back to Frame 46). Then, the above equation can be written as

$$SS_{between} = \frac{1}{n} \sum_{j=1}^{k} (T_j)^2 - \frac{(\sum X)^2}{N}$$

$$= \frac{1}{n} [\quad\quad\quad] - \frac{(\sum X)^2}{N}$$

$$n \sum_{j=1}^{k} \left(\frac{\sum_{i=1}^{n} X_{ij}}{n} \right)^2$$

$$- 2n \left(\frac{\sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij}}{nk} \right) \sum_{j=1}^{k} \left(\frac{\sum_{i=1}^{n} X_{ij}}{n} \right)$$

$$+ nk \left(\frac{\sum_{j=1}^{k} \sum_{i=1}^{n} X_{ij}}{nk} \right)^2$$

(Notice, $\bar{X}_t$ is a constant.)

---

(51) It must be pointed out that the term $\frac{(\sum X)^2}{N}$, that is the value of $\frac{(\text{grand total})^2}{\text{total number of cases}}$, is a very important quantity in obtaining sums of squares, SS's, in the analysis of variance. You have used this term in computing

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} \quad \text{and}$$

$$SS_{between} = \frac{1}{n}[(T_1)^2 + (T_2)^2 + \ldots + (T_k)^2] - \frac{(\sum X)^2}{N}$$

As mentioned in Frame 9, this important term is referred to as the ________________.

$$(T_1)^2 + (T_2)^2 + \ldots + (T_k)^2$$

---

(52) We have determined SS_{total} (Frames 34 through 40) and $SS_{between}$ (Frames 41 throught47). Now we are ready to obtain SS_{within} (the within-treatment sum of squares). Since the total sum of squares is partitioned into (1) the between-treatment sum of squares and (2) the within-treatment sum of squares, that is,

$$SS_{total} = SS_{between} + SS_{within} \quad \ldots \ldots \quad \text{(II-5)}$$

Substituting in this formula the value of SS_{total} from Frame 37 (or 39), and the value of $SS_{between}$ from Frame 45 (or 47), we can compute the value

$$SS_{within} = SS_{total} - SS_{between}$$

$$= \_\_\_\_\_ - \_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_$$

correction term (or correction factor)

(53) Substituting Formula II-1 (Frame 38) and Formula II-4 (Frame 46) in Formula II-5 shown above, you may still have another computational formula:

$$SS_{within} = SS_{total} - SS_{between}$$

$$= \left\{\sum X^2 - \frac{(\sum X)^2}{N}\right\} - \left\{\frac{1}{n}\left[(T_1)^2 - (T_2)^2 - \ldots - (T_k)^2\right] - \frac{(\sum X)^2}{N}\right\}$$

$$SS_{within} = \sum X^2 - \frac{1}{n}\left[(T_1)^2 + (T_2)^2 + \ldots + (T_k)^2\right] \quad \text{- - - - - - - - - (II-6)}$$

In this formula the ________________ term is eliminated.

101.5 - 20.6

80.9

---

(54) Graphically, we may determine SS_{within} as follows: In determining the within-treatment sum of squares we are concerned only with the variation within the three individual treatments. So we would, first, obtain deviation of each coded score from its own treatment mean. This is done by subtracting from each score in the first column, Table 2-c (Frame 35) its treatment mean $\bar{X}_1 = -0.4$; from each score in the second column $\bar{X}_2 = 1.6$; from each score in the third column $\bar{X}_3 = 0.3$. The results are then shown in a table like this:

Table 2-1 $X - \bar{X}_j$

	Treat. A $X - \bar{X}_1$	Treat. B $SX - \bar{X}_2$	Treat. C $X - \bar{X}$	Total
	-0.6	1.4	3.7	
	1.4	0.4	0.7	
	-0.6	-0.6	-0.3	
	2.4	-1.6	-1.3	
	0.4	-0.6	-2.3	
	-1.6	2.4	0.7	
	1.4	0.4	1.7	
	-2.6	-2.6	-0.3	
	-1.6	-1.6	-2.3	
	1.4	2.4	-0.3	
Sum	0.0	0.0	0.0	0.0

Notice that the sum of deviations is equal to zero for each treatment. $\sum x = 0$ can be proved as follows;

$$x = \sum(X - \bar{X})$$
$$= \sum X - \sum \bar{X}$$
$$= \sum X - \_\_\_\_$$
$$= \sum X - \_\_\_\_$$
$$= 0$$

correction [the correction term $\frac{(\sum X)^2}{N}$]

(55) Now, we may multiply each item in the above table by itself and sum up all the squared deviations to obtain the within-treatment sum of squares. The following table gives the results:

Table 2-m $(X-\bar{X}_1)^2$ & $\sum(X-\bar{X}_j)^2$

	Treat. A $(X-\bar{X}_1)^2$	Treat. B $(X-\bar{X}_2)^2$	Treat. C $(X-\bar{X}_3)^2$	Total $(X-\bar{X}_j)^2$
	0.36	1.96	13.69	
	1.96	0.16	0.49	
	0.36	0.36	0.09	
	5.76	2.56	1.69	
	0.16	0.36	5.29	
	2.56	5.76	0.49	
	1.96	0.16	2.89	
	6.76	6.76	0.09	
	2.56	2.56	5.29	
	1.96	5.76	0.09	
Sum	24.40	26.40	30.10	80.90

Thus, we have SS_{within} = ________________

(56) Let us conclude the step of determining the sums of squares by entering the values we have obtained in our table of ANOVA as follows:

Table 3-b Sum of Squares

Source of variation	Sum of Squares	df	s^2
Between T.	20.6		
Within T.	80.9		
Total	101.5		

The next step in the analysis of variance is to determine the ________________ for (1) total sum of squares, (2) between-treat. sum of squares, and (3) within-treat. sum of squares.

$n\bar{X}$

$n\,\frac{\Sigma X}{n}$

80.90

(57) In ANOVA, the number of degrees of freedom, df, may be regarded as the number of deviations, $(X-\bar{X})$, in the sum of squares, $\sum(X-\bar{X})^2$, that are free to vary. As we have observed that the sum of deviations in a frequency distribution is always equal to zero. That is, $\sum(X-\bar{X}) = 0$. Suppose we have three scores X_1, X_2, and X_3 which yield a mean $\bar{X}$, then

$$(X_1-\bar{X}) + (X_2-\bar{X}) + (X_3-\bar{X}) = 0$$

Now, we can select any value for two scores, say, X_1 and X_2, but once X_1 and X_2 take definite values, the value of X_3 is fixed. That is, in this case there are two deviations that are ______________ to vary.

degrees of freedom
(or df)

---

(58) In other words, any two out of the three scores can be regarded as independent variables (this is to say that they vary freely) and the third score as a dependent variable (this is to say that its value is fixed once the other two scores take definite values). So, the number of deviations that are free to vary, in this case, is __________. Therefore, the sum of squares has ______________ degrees of freedom.

free

---

(59) To generalize this observation, let us consider a sum of squares of deviations, $\sum(X-\bar{X})^2$, whose mean is computed by N scores. Since $\sum(X-\bar{X}) = 0$,

$$(X_1-\bar{X}) + (X_2-\bar{X}) + \ldots + (X_n-\bar{X}) = 0$$

Mean $\bar{X}$ is a constant. Thus, as soon as the values of any N-1 scores are selected the value of the remaining score is ____________. Therefore, the number of degrees of freedom df = _____.

2

2

(60) This is equivalent to saying that there are ____________ deviations which vary freely, or that any one deviation is fixed as soon as other (N-1) deviations take definite values and, thus, lose __________ degree of freedom. This restriction is caused because we are taking deviations about one constant, the mean $\overline{X}$, computed from N scores at hand. Enlarging this reasoning further, the number of degrees of freedom for the sum of squares may also be regarded as the number of squares minus the number of restrictions imposed by constants computed from the data.

fixed

N-1

---

(61) The total sum of squares, df_{total}, is computed from N scores. Thus,

$$df_{total} = N - 1 \ldots\ldots\ldots\ldots (II-7)$$

where N = total number of cases
= nk
n = number of cases in each treatment
k = number of treatments

In our example, we have 30 cases (10 cases each for 3 treatments). Therefore,

$$df_{total} = \_\_\_\_\_\_\_\_\_\_\_\_$$

N-1

1

---

(62) To obtain the number of degrees of freedom for the between-treatment sum of squares, we subtract one from the number of treatments, k. That is,

$$df_{between} = k - 1 \ldots\ldots\ldots\ldots (II-8)$$

Since we have 3 treatments in our example,

$$df_{between} = \_\_\_\_\_\_\_\_\_\_\_\_$$

30 - 1 = 29

(63) The total degrees of freedom can also be partitioned into (1) the between-treatment degrees of freedom and (2) the within-treatment degrees of freedom. That is,

$$df_{total} = df_{between} + df_{within} \quad . \ . \ . \ . \ \text{(II-9)}$$

Therefore, in our example

$$df_{within} = df_{total} - df_{between}$$

$$= \underline{\qquad\qquad} - \underline{\qquad\qquad}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

3 - 1 = 2

---

(64) Or, we may argue like this. Since each treatment consists of n cases, the number of degrees of freedom within each treatment is n-1. There are k such treatments, hence

$$df_{within} = k(n-1) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ \text{(II-10)}$$

In our case,

$$df_{within} = \underline{\qquad\qquad\qquad\qquad}$$

To conclude this stage of the analysis, let us enter the values of df's obtained in our Table of ANOVA as follows:

Table 3-c Degrees of Freedom

Source of variation	Sum of squares	Degree of freedom	s^2	F
Between T.	20.6	2		
Within T.	80.9	27		
Total	101.5	29		

29 - 2

27

---

(65) The next step is to compute two variance estimates: one based on the between-treatment variation, the other based on the within-treatment variation. When we divide the sum of squares by the corresponding number of degrees of freedom on which the sum of squares is based, we obtained the variance estimate which is often referred to as the mean square, denoted by M. S.

Thus, the between-treatment variance estimate (or mean square) is obtained by

$$\underline{s^2_{between}} = MS_{between} = \frac{SS_{between}}{df_{between}} \quad . \ . \ . \ . \ \text{(II-10)}$$

In our example, thus

$$s^2_{between} = \underline{\qquad\qquad\qquad\qquad}$$

3(10-1) = 27

(66) The within-treatment variance estimate (or mean square) is obtained by

$$s^2_{within} = MS_{within} = \frac{SS_{within}}{df_{within}} \; . \; . \; . \; . \; . \; . \quad (II-11)$$

Thus, in our example,

s^3_{within} = ____________

Our Table of ANOVA now lookslike this:

Table 3-d Variance Estimate

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F
Between T.	20.6	2	10.300	
Within T.	80.9	27	2.996	
Total	101.5	29		

$\frac{20.6}{2} = 10.300$

---

(67) If we set up a null hypothesis which assumes that there is no difference between the between-treatment variation and the within-treatment variation, we are assuming, in effect, that the two variance estimates $s^2_{between}$ and s^2_{within} are estimates of the ____________ population variance.

$\frac{80.9}{27} = 2.996$

---

(68) There are two kinds of hypotheses: (1) research hypotheses and (2) statistical hypotheses. A research hypothesis is postulated on the basis of theory or prior observation or on logical grounds and is usually concerned with a prediction in regard to the outcome of an experiment. A statistical hypothesis is usually known as a null hypothesis which is set up for the purpose of testing the research hypothesis. A null hypothesis is a ____________ ____________ hypothesis, usually denoted by H_o. In an experiment, the postulated null hypothesis is tested against the alternative hypothesis (or hypotheses), denoted by H_1 (or H_1, H_2, etc.), which is (or are) the ____________ hypothesis (or hypotheses).

same

(69) The next and final step is to test the null hypothesis which assumes that there is no difference between the two variance estimates, $s^2_{between}$ and s^2_{within} through the use of the F (or variance ratio)-test. The within-treatment variance estimate, s^2_{within}, in the completely randomized design is considered as the experimental error against which the significance of the observed difference may be tested. It measures the uncontrolled variation between individual subjects treated alike, and thus corresponds to a standard error of a difference as used in the t-test. The F-ratio for the completely randomized design is, thus, obtained by:

$$F = \frac{s^2_{between}}{s^2_{within}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (II\text{-}12)$$

Therefore, in our example, F = ____________________

statistical

research

---

(70) We now compare the magnitude of the observed variance ratio $F_{ob} = 3.44$ with the critical F values at the 5%, 1%, and 0.1% levels, based on the same numbers of degrees of freedom, given by the Table of F provided by any statistics textbook. If the value of F_{ob} exceeds the corresponding critical values, we shall conclude that the F_{ob} is statistically significant and we shall reject the null hypothesis. If the value of Fob does not exceed the critical values, we shall conclude that the F_{ob} is statistically ______________ and we shall ______________ the null hypothesis. When $F_{.95} < F_{ob} < F_{.99}$, the observed F-ratio is said to be significant at the 5% level. When $F_{.99} < F_{ob} < F_{.999}$, the observed F-ratio is said to be significant at 1% level. Notice that $F_{.95}$, $F_{.99}$, and $F_{.999}$ represent the tabled critical values at 5%, 1%, and 0.1% levels based on the same numbers of degrees of freedom, respectively. The symbol $<$ means "is less than".

$$\frac{10.300}{2.996} = 3.44$$

(71) Let us look up the critical F values in the Table of F provided by your textbook. Notice that the columns of the Table are concerned with the n_1's, the degrees of freedom for the greater variance estimate, and that the rows have to do with the n_2's, the degrees of freedom for the lesser variance estimate. Our greater variance estimate $s_{between}$ has 2 degrees of freedom and our lessor variance estimate s_{within} has 27 degrees of freedom, and we should check column (denoted by n_1) __________ and row (denoted by n_2) ______________ of the Table of F for the critical F values, at 5%, 1%, and 0.1% levels.

not significant

not reject

---

(72) A portion of the Table of F is shown below. From this table, we obtain the critical values:

Table 4 F's for 5%, 1%, & 0.1% Levels

n_2 \ n_1	1	2	3
26	4.22 7.22 13.74	3.37 5.53 9.12	2.98 4.64 7.36
27	4.21 7.68 13.62	3.35 5.49 9.02	2.96 4.60 7.27

$F_{.95(2,27)} = 3.35$, $F_{.99(2,27)} = 5.49$, and $F_{.999(2,27)} = 9.02$. Since $F_{ob} = 5.44$, $F_{.95(2,27)} < F_{ob\,(2,27)} < F_{.99(2,27)}$. Thus, the observed F-ratio is statistically significant at ______% level. This is equivalent to saying that the null hypothesis is ________, that is, that the two variance estimates are significantly ____________ at ________% level.

2

27

---

(73) We have just rejected, at P = 0.05 level, the ________ ______ which assumes that there is no difference existing between the variance estimates $s^2_{between}$ and s^2_{within} other than that due to chance alone. We may now conclude by saying that the observed variance ratio, F_{ob}, is too great for the null hypothesis to be maintained and that the difference among the three treatments is statistically ______________.

5

rejected

different

5

(74) Now, let us conclude our analysis by entering the observed value of F with a symbol indicating the level of significance in the Table of Analysis of Variance like this:

Table 3-e F-Ratio

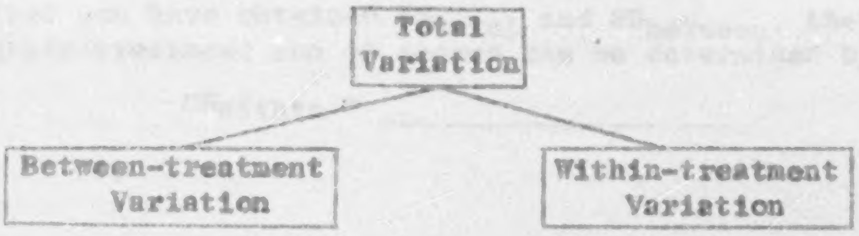
Source of variation	Sum of squares	Degree of freedom	Variance estimate	F-ratio
Between T.	20.6	2	10.300	3.44*
Within T.	80.9	27	2.996	
Total	101.5	29		

Conventionally, one asterick, *, is used to indicate the 5% level of significance; two asterisks, **, indicate significance at the 1% level; and three asterisks, ***, denote the 0.1% level of significance. $F_{ob}=3.44$ is significant at ______% level. Therefore, the value 3.44 should be marked with ________________.

---

(75) Summary In the completely randomized design, with equal n's the total variation is partitioned into two __________ components: (1) the between-treatment variation and (2) the within-treatment variation.

Figure 1 Partitioning of Total Variation -Completely Randomized Design (Equal n's)



null hypothesis

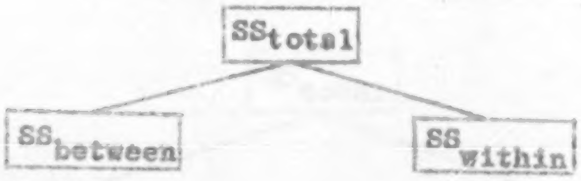
(parent) population

5

* (one asterisk)

(76) The total sum of squares, SS_{total}, is partitioned into (1) the between-treatment sum of squares, $SS_{between}$, and (2) the within-treatment sum of squares, SS_{within}.

Figure 2 Partitioning of the Total Sum of Squares - CRD (Equal n's)



Let X = raw (or coded) score
N = total number of cases in the experiment

Then the total sum of squares can be obtained by (raw score method):

$$SS_{total} = \sum X^2 - \_\_\_\_\_\_\_\_$$

independent

---

(77) Let n = number of cases in each treatment
k = number of treatments
T_k = kth treatment total

Then, the between sum of squares, by raw score method, can be obtained by

$$SS_{between} = \_\_\_\_\_\_\_\_ - \frac{(\sum X)^2}{N}$$

After you have obtained SS_{total} and $SS_{between}$, the within-treatment sum of scores can be determined by

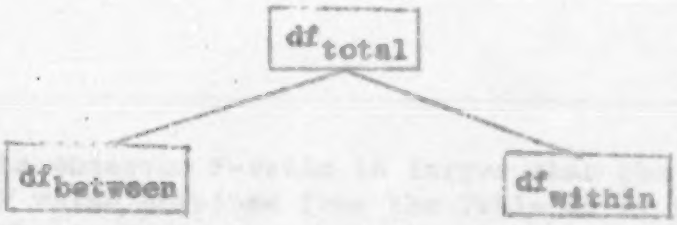
$$SS_{within} = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\frac{(\sum X)^2}{N}$$

(78) The total number of degrees of freedom, df_{total}, can also be partioned into (1) the between-treatment degrees of freedom, $df_{between}$, and (2) the within-treatment degrees of freedom, df_{within}. That is,

df_{total} = ____________________

Figure 3 Partitioning of the Total Degrees of Freedom - CRD (Equal n's)



$\frac{1}{n}[(T_1)^2+(T_2)^2+\cdots+(T_k)^2]$

$SS_{total}-SS_{between}$

---

(79) Knowing the total number of cases, N, the total degrees of freedom can be obtained by

df_{total} = ________________

Knowing the number of treatments, k, the between-treatment degrees of freedom can be determined by

$df_{between}$ = ________________

Knowing df_{total} and $df_{between}$, we have the within-treatment degrees of freedom

df_{within} = ________________

$df_{between} + df_{within}$

---

(80) The variance estimate in ANOVA is usually referred to as the ____________. Two variance estimates are to be computed: (1) the between-treatment variance estimate and (2) the within-treatment variance estimate. To get the variance estimate, the sum of squares is divided by its corresponding ______________. Thus, the between-treatment variance estimate and the within-treatment variance are obtained by

$s^2_{between}$ = ______________

s^2_{within} = ______________

N - 1

K - 1

$df_{total} - df_{between}$

(81) The within-treatment variance estimate, s^2_{within}, is considered to be the __________ variance against which the between-treatment variance estimate, $s^2_{between}$, may be tested. Thus, the observed F-ratio is obtained by F_{ob} = ______________	mean square (or MS) degrees of freedom $\frac{SS_{between}}{df_{between}}$ $\frac{SS_{within}}{df_{within}}$
(82) If the observed F-ratio is larger than the critical F value obtained from the Table of F, the null hypothesis is __________. That is, there is significant ____________ among the treatments. If the observed F-ratio is smaller than the critical F value, we conclude that there is ______________ difference among the treatments other than that due to chance alone.	error $\frac{s^2_{between}}{s^2_{within}}$

II. 2. Completely Randomized Design with Unequal n's

(83) Suppose that we have a total of 23 individual subjects with 6, 7, and 10 subjects assigned to treatments I, II, and III at random. This is a completely randomized design with unequal n's. This differs from the completely randomized design with equal n's in that we do not have ______ number of cases for each treatment any more.

rejected

difference

no significant

---

(84) Assume that the following data have been observed.

Table 5-a Observed Data (II)

Treat. I $n_1 = 6$	Treat. II $n_2 = 7$	Treat. III $n_3 = 10$
18	24	35
27	35	40
26	29	29
30	36	19
19	38	37
24	27	34
	13	29
		31
		28
		33

First of all, we may or may not ______ the observed data so as to simplify the computation that will follow.

equal

---

(85) In coding the observed data it is suggested that from each score a constant which is near the grand mean in magnitude be subtracted. In obtaining the following table from Table 5-a (Frame 84) a constant ______ was used.

Table 5-b Coded Form (II)

Treat. I $n_1 = 6$	Treat. II $n_2 = 7$	Treat. III $n_3 = 10$
-7	-1	8
2	10	15
1	4	4
5	11	-6
-6	13	12
-1	2	9
	-7	4
		6
		3
		8

code

(86) The next step is to ____________ the total variation into two independent components of variation: (1) the between-treatment variation and (2) the ____________ variation. Thus, our Table of ANOVA looks like this:

Table 6-a Source of Variation

Source of Variation	ss	df	s^2	F
Between T. Within T.				
Total				

25

---

(87) To obtain the sums of squares, SS's, let us construct a table in which both X's and X^2's appear. This can be done by adding new columns of X^2's to Table 5-b (Frame 85).

Table 5-c n_j, X, & X^2

Treat. I		Treat. II		Treat. III		Total	
$n_1 = 6$		$n_2 = 7$		$n_3 = 10$		N = 23	
X	X^2	X	X^2	X	X^2	X	X^2
-7	49	-1	1	8	64		
2	4	10	100	15	225		
1	1	4	16	4	16		
5	25	11	121	-6	36		
-6	36	13	169	12	144		
-1	1	2	4	9	81		
		-7	49	4	16		
				6	36		
				3	9		
				8	64		

$$N = n_1 + n_2 + \ldots + n_k \quad . \; . \quad \text{(II-13)}$$

where N = total number of cases

n_k = number of cases in the kth treatment

In our case,

$$N = n_1 + n_2 + n_3 = \underline{\qquad\qquad}$$

partition

within-treatment

(88) Summing the six columns in the above table, and totaling the column sums, we have

Table 5-d $\sum X$ & $\sum X^2$

	Treat. I $n_1 = 6$ X	 X^2	Treat. II $n_2 = 7$ X	 X^2	Treat. III $n_3 = 10$ X	 X^2	Total N = 23 X	 X^2
	-7	49	-1	1	8	64		
	2	4	10	100	15	225		
	1	1	4	16	4	16		
	5	25	11	121	-6	36		
	-6	36	13	169	12	144		
	-1	1	2	4	9	81		
			-7	49	4	16		
					6	36		
					3	9		
					8	64		
Sum	**-6**	**116**	**32**	**460**	**63**	**691**	**89**	**1267**

Let $\sum X$ = sum of all the coded scores
$\sum X^2$ = sum of all the squared scores.
Then, *from* the table we have

$\sum X$ = ________
$\sum X^2$ = ________

---

(89) The total sum of squares is computed by the same formula we have used for the equal n's design. That is,

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

where $\frac{(\sum X)^2}{N}$ is referred to as the ________. Substituting the necessary values from Table 5-d we, thus, have

SS_{total} = ________

---

(90) The formula used for computing the between sum of squares, $SS_{between}$, is different from that used for the equal n's design. For the unequal n's design we use

$$SS_{between} = \left[\frac{(T_1)^2}{n_1} + \frac{(T_2)^2}{n_2} + \ldots + \frac{(T_k)^2}{n_k}\right] - \frac{(\sum X)^2}{N} \quad \ldots\ldots\ldots \text{(II-14)}$$

where T_k = kth treatment total
$\sum X$ = grand total
n_k = number of cases in the kth treatment
N = total number of cases

Therefore, substituting the necessary values from Table 5-d (Frame 88), we have

$SS_{between}$ = ________
= ________

6 + 7 + 10 = 23

89

1267

Correction term

$1267 - \frac{(89)^2}{23} = 1267 -$
= 922.61

(91) The total sum of squares is _____ into (1) the between-treatment sum of squares and (2) the within-treatment sum of squares. That is, $SS_{total} = SS_{between} + SS_{within}$ Thus, substituting the values of SS_{total} and $SS_{between}$ we have just computed in the equation, we have $SS_{within} =$ ________ $-$ ________ $=$ ________________	$[\frac{(-6)^2}{6} + \frac{(32)^2}{7} + \frac{(63)^2}{10}] - \frac{(89)^2}{23}$ $= 204.80$
(92) The number of degrees of freedom for the total sum of squares is one less than the total number of cases. This is so because ________ restriction existed when we computed the total sum of squares based on the total number of cases. Thus, $df_{total} = N - 1 =$ ____________	partitioned $SS_{total} - SS_{between}$ $= 922.81 - 204.80$ $= 717.81$
(93) The number of degrees of freedom for the between-treatment sum of squares is equal to the number of ________ minus one. Again, this is so because we computed the between-treatment sum of squares based on k treatment means. Thus, $df_{between} = K - 1 =$ ____________	one $23 - 1 = 22$
(94) The total degrees of freedom is partitioned into (1) the __________ degrees of freedom and (2) the within-treatment degrees of freedom. That is, $df_{total} = df_{between} + df_{within}$ Substituting the values of df_{total} and $df_{between}$ we have just computed, we have $df_{within} =$ ________ $-$ ________ $=$ ________________	treatments $3 - 1 = 2$

(95) Or, we may consider the within-treatment degrees of freedom, df_{within}, as the pooled degrees of freedom within each treatment. That is,

$$df_{within} = (n_1-1)+(n_2-1)+\ldots\ldots (n_k-1) \qquad \ldots\ldots (II\text{-}15)$$

In our example, we have ________________ treatments. Substituting the n's in the equation we, then, have

$$df_{within} = \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

between-treatment

$$df_{total} - df_{between}$$
$$= 22 - 2 = 20$$

---

(96) The variance estimate is obtained by dividing the sum of squares by its corresponding ________________. Thus,

$$s^2_{between} = \frac{SS_{between}}{df_{between}} = \_\_\_\_\_\_\_\_\_\_,$$

$$s^2_{within} = \frac{SS_{within}}{df_{within}} = \_\_\_\_\_\_\_\_\_\_,$$

3

$$(n_1-1)+(n_2-1)+(n_3-1)$$
$$= (6-1)+(7-1)+(10-1)$$
$$= 20$$

---

(97) Let us collect the results of the computation so far in the Table of Analysis of Variance. We Have,

Table 6-b ANOVA (II)

Source of Variation	Sum of Squares	Degree of freedom	Variance Estimate	F-ratio
Between S.	204.80	2	102.40	
Within S.	717.81	20	35.89	
Total	922.61	22		

The variance ratio is

$$F_{ob} = \frac{s^2_{between}}{s^2_{within}} = \_\_\_\_\_$$

number of degrees of freedom

$$\frac{204.80}{2} = 102.40$$

$$\frac{717.81}{20} = 35.89$$

$$\frac{102.40}{35.89} = 2.85$$

(98) To test the significance of difference between the two variance estimates, we must consult the Table of F. Our greater variance estimate $s^2_{between}$ has ________ degrees of freedom and the lesser variance estimate s^2_{within} has ________ degrees of freedom. Thus, we should check column ______ and row ______ of the tables of F for the critical values. Table 7 shows a portion of the table. From this table, we

TABLE 7 - F's for 5%, 1%, and 0.1% levels

n_2 \ n_1	2	12	24
2		19.41 99.42 999.4	19.45 99.46 999.5
20	3.49 5.85 9.95		

obtain $F_{.95(2,20)}$ = ______________;

$F_{.99(2,20)}$ = ______________, and

$F_{.999(2,20)}$ = ______________ .

2

20

2 20

3.49

5.85

9.95

---

(99) Finally, we compare $F_{ob} = 2.85$ with these critical values. We find that the observed F-ratio does not exceed these critical values. Therefore, the null hypothesis is ________________. This is equivalent to saying that there is no statistically significant difference among the three treatments.

not rejected

---

(100) Summary In the completely randomized unequal n's design, the total variation is partioned into two ________________ components: (1) the between-treatment variation and (2) the within-treatment variation. This is also true for the equal n's design.

(101) The total sum of squares, SS_{total}, is partitioned into (1) the ________ sum of squares and (2) the ________ sum of squares. The total sum of squares is obtained by

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

where $\sum X^2$ = grand total of each score squared
$\sum X$ = grand total of all the scores
N = ________

We also use the same formula for the equal n's design.

independent

---

(102) The formula used for computing the between-treatment sum of squares is different from that used for the equal n's design. The formula is

$$SS_{between} = \left[\frac{(T_1)^2}{n_1} + \frac{(T_2)^2}{n_2} + \ldots + \frac{(T_k)^2}{n_k}\right] - \frac{(\Sigma X)^2}{N}$$

where T_k = the k th treatment ________
$\frac{(\Sigma X)^2}{N}$ = the ______ term.

between-treatment

within-treatment

total number of cases

---

(103) We know that

$$SS_{total} = SS_{between} + SS_{within}$$

Thus,

$$SS_{within} = SS_{total} - SS_{between}$$

Or,

$$SS_{within} = \underline{(\qquad) - (\qquad)}$$

$$= \sum X^2 - \left[\frac{(T_1)^2}{n_1} + \frac{(T_2)^2}{n_2} + \ldots + \frac{(T_k)^2}{n_k}\right]$$

total

correction

---

(104) Let N = total number of cases
k = number of treatments
n_k = number of cases in the kth treatment

Then, $df_{total} = (n_1 + n_2 + \ldots\ldots + n_k) - 1$
$= \underline{\qquad\qquad\qquad} - 1$
$df_{between} = k - 1$
$df_{within} = df_{total} - df_{between}$
$= (n_1 + n_2 + \ldots + n_k - 1) - (k - 1)$
$= n_1 + n_2 + \ldots + n_k - k$
$= \underline{\qquad\qquad\qquad} - k$

$$\left\{\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right\} - \left\{\left[\frac{(T_1)^2}{n_1} + \frac{(T_2)^2}{n_2} + \ldots + \frac{(T_k)^2}{n_k}\right] - \frac{(\Sigma X)^2}{N}\right\}$$

N

N

(105) Two variance estimates are to be computed. One is the ________________ variance estimate, the other is the ____________ variance estimate. The variance estimate which is often called the __________ is obtained by dividing the __________ by its corresponding ________________. Thus,

$$s^2_{between} = \frac{SS_{between}}{df_{between}}$$

$$s^2_{within} = \frac{SS_{within}}{df_{within}}$$

---

between-treatment (or within-treatment)
within-treatment (or between-treatment)

mean square (MS)
sum of squares (SS)
degrees of freedom (df)

(106) The observed F-ratio is obtained by dividing the ______________ variance estimate (or mean square) by the __________ variance estimate (or mean square). That is,

$$F_{ob} = \frac{s^2_{between}}{s^2_{within}}$$

The within-treatment variance estimate is considered to be the ____________ variance estimate here.

---

between-treatment

within-treatment

error

(107) Finally, the observed F-ratio is compared with the critical F values. To do this, you have to look up the Tables of F for the critical values. The number of degrees of freedom for between-treatment variance estimate, $df_{between}$, gives you the number of _____________ of the F table, and the number of degrees of freedom for within-treatment variance estimate, dfwithin, gives you the number of _______ of the Tables F in which the critical value is to be found.

---

column

row

(108) If the observed F ratio exceeds the critical F value, the null hypothesis is ____________. You will, then, conclude by saying that there is significant ________ among the treatment means.

II. 3. Randomized Block Design

(109) The use of blocking arose in comparative experiments in agricultural research where the terms used in describing the designs had real physical meaning. The experimental area was used to mean an area—a piece of ground. A block was a smaller piece of ground, small enough to be fairly uniform in soil and topography. A plot was an even smaller piece of ground which is the basic unit (experimental unit) of the design. We are now ready to discuss the block designs, an important type of experimental designs, in which the experimental unit is called the ____________.	rejected difference
(110) Suppose that we are concerned with the effects of four different treatments of a plastic material. In order to achieve a good comparison of the four treatment effects, knowing that plastic properties vary considerably within a given sheet, we would divide the plastic sheet into more or less homogeneous areas, and then subdivide each area into four parts on which the four different treatments may be applied. According to the terminology described in (109), each of the sub-divided parts, the smallest units, provides a __________; and each set of four smallest units forms a __________.	plot
(111) In industrial and engineering research, the blocking can be used to take advantage of naturally homogeneous groupings in materials, machines, times, etc. so as to balance out some background variables which are not directly factors (experimental variables) in the experiment. For example, in comparing the wearing qualities of plastic soles for shoes, the natural block is a block of two, __________ of an individual. In comparing the wearing qualities of automobile tires, the natural block is a block of __________, the __________ of an automobile. Each automobile may travel over different terrain or have different drivers. The four tires on any given automobile, however, will undergo much the same condition, especially if they are rotated frequently.	plot block

(112) In educational and psychological research, the experimental unit corresponding to a plot is usually a subject. A group of subjects more or less homogeneous with respect to some variable is equivalent to a ________________.	the two feet four four wheels
(113) In general, a randomized block design is one in which each of the treatments appears exactly once and only once in every block, and the treatments are randomly allocated to the ________________ (experimental units) within a given block.	block
(114) It must be mentioned that the randomized block design is known as the "matched group design" in educational and psychological research. This is due to the fact that in this specific type of design, each block of subjects is matched with respect to a given varible which can improve ________________ in the absence of the treatment effects. This is why the randomized block (or matched group) design can provide for a more sensitive test than the completely randomized design.	plots

(115) The results of a randomized block experiment are usually given by a two-way table like this:

homogeneity

Table 8 Randomized Block Design

Block	Treatment				Total
	1	2	....	k	
1					B_1
2					B_2
.					.
.					.
.					.
n					B_n
Total	T_1	T_2	....	T_k	ΣX

where T_k = kth treatment total
B_n = nth block total
ΣX = ____________________

---

(116) Suppose that a total of 30 subjects have been grouped into 10 blocks of three subjects each, on the basis of an initial measure having to do with a background variable. Three subjects within each block were then assigned to three different treatments at random, resulted with the following data. This is a ______________ experiment.

grand total

Table 8-a Observed Data (III)

Block	Treatment 1	Treatment 2	Treatment 3
1	26	25	23
2	25	24	23
3	25	24	21
4	24	23	20
5	22	21	19
6	22	19	18
7	20	19	18
8	18	18	17
9	16	17	16
10	14	15	13

(117) To obtain the coded form Table 8-b shown below each score in Table 8-a was added a number of ________.

Table 8-b X, X^2, ΣX, & ΣX^2 (III)

Block	Treat 1		Treat. 2		Treat. 3		Total	
	X	X^2	X	X^2	X	X^2	X	X^2
1	6	36	5	25	3	9	14	70
2	5	25	4	16	3	9	12	50
3	5	25	4	16	1	1	10	42
4	4	16	3	9	0	0	7	25
5	2	4	1	1	-1	1	2	6
6	2	4	-1	1	-2	4	-1	9
7	0	0	-1	1	-2	4	-3	5
8	-2	4	-2	4	-3	9	-7	17
9	-4	16	-3	9	-4	16	-11	41
10	-6	36	-5	25	-7	49	-18	110
Total	12	166	5	107	-12	102	5	375

From this table, we obtain the correction term =

$$\frac{(\Sigma X)^2}{N} = \_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_ .$$

randomized block (or matched group)

---

(118) Substituting this value and the value of ΣX^2 in the following formula, we have the total sum of squares

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} \;.\;.\;.\;.\;.\;.\; \text{(II -16)}$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

-20

$$\frac{(5)^2}{30} = 0.83$$

---

(119) The total variation in the randomized block experiment is partitioned into three components: (1) between-treatment variation, (2) between-block variation, and (3) residual variation. Thus, the next step is to compute the between-treatment sum of squares:

$$SS_{treat} = \frac{1}{n}[(T_1)^2+(T_2)^2+(T_3)^2] - \frac{(\Sigma X)^2}{N} \cdot\cdot \text{(II -17)}$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

375 - 0.83 = 374.17

(120) The computational formula used in calculating the between-block sum of squares is similar to that used for the between-treatment sum of squares.

$$SS_{block} = \frac{1}{k}\left[(B_1)^2+(B_2)^2+\ldots+(B_n)^2\right]-\frac{(\Sigma X)^2}{N} \quad \ldots(II\text{-}18)$$

= ______________________

$$\frac{1}{10}\left[(12)^2+(5)^2+(-12)^2\right]-0.83$$

$= 30.47$

---

(121) The residual variation may be considered as the treatment-block interaction (to be discussed later in the factorial design) which in this case is taken as the experimental error. The realtionship of the sum of squares is given by

$$SS_{total} = SS_{treat} + SS_{block} + SS_{residual} \ldots (II\text{-}19)$$

Substituting the necessary values in this formula, we have the residual sum of squares:

SS_{total} = ________ - __________ - __________

= ______________

$$\frac{1}{3}\left[(14)^2+(12)^2+(10)^2+(7)^2+(2)^2+(1)^2+(-3)^2+(-7)^2+(-11)^2+(-18)^2\right]-0.83$$

$= 332.33 - 0.83$

$= 331.50$

---

(122) The number of degrees of freedom for the total sum of squares is one less than the ______________

That is, $df_{total} = N - 1$ . . . . (II-20)

Substituting the value of N, in the formula, we have

df_{total} = ______________

$SS_{total} - SS_{treat} - SS_{block}$

$= 374.17-30.47-331.50$

$= 12.20$

(123) The number of degrees of freedom for the between-treatment sum of squares is equal to the number of treatments, k, minus __________. This is so because there are __________ treatment means which can vary freely when we compute the between-treatment __________.

$df_{treat} = k - 1$ . . . . (II-21)

Substituting the value of k in the formula, we have

$df_{treat} =$ _______________

total number od cases (or subjects)

30 - 1 = 29

---

(124) Based on the similar reasoning,

$df_{block} = n - 1$ . . . . (II-22)

where n stands for the number of blocks in the experiment. Therefore, in this case

$df_{block} =$ ______________.

one

k-1

sum of squares

3 - 1 = 2

---

(125) The total number of degrees is __________ into (1) the between-treatment degrees of freedom, (2) the between-block degrees of freedom, and (3) the residual degrees of freedom. That is,

$df_{total} = df_{treat} + df_{block} + df_{residual}$ . . (II-23)

Substituting the necessary values, we then have the residual degrees of freedom:

$df_{residual} =$ ________ - ________ - ________

$=$ ______________

10 - 1 = 9

(126) Now, we may compute three variance estimates designated s^2. The variance estimate which is also known as the ____________ can be obtained by dividing the ____________ by its corresponding number of degrees of freedom. That is:

$$s^2_{treat} = \frac{SS_{treat}}{df_{treat}} \quad . . . . (II-24)$$

$$s^2_{block} = \frac{SS_{block}}{df_{block}} \quad . . . . (II-25)$$

$$s^2_{residual} = \frac{SS_{residual}}{df_{residual}} \quad . . . . (II-26)$$

Substituting the values of SS's and df's, we have

s^2_{treat} = ____________

s^2_{block} = ____________

$s^2_{residual}$ = ____________

partitioned

$df_{total} - df_{treat} - df_{block}$

$= 29 - 2 - 9$

$= 18$

---

(127) The blocks do not constitute a factor in our present example. That is, we are not interested in testing the significance of difference among the ten blocks. Rather, we are concerned with the effects among the three treatments. We would test the between-treatment variance estimate against the residual variance estimate which is considered as the error variance. This gives the observed F-ratio. That is,

$$F_{ob} = \frac{s^2_{treat}}{s^2_{residual}} \quad . . . . (II-27)$$

Substituting the necessary values in the formula, we have

F_{ob} = ____________

mean square (or MS)

sum of squares

$\frac{30.47}{2} = 15.24$

$\frac{331.50}{9} = 36.83$

$\frac{12.20}{18} = 0.68$

(128) To look up the critical F value in the Table of F, you would check Column ______, Row ______ of the Table because the number of degrees of freedom for the numberator of Formula II -27 is $df_{treat} = 2$, the number of degrees of freedom for the denominator is $df_{residual} = 18$. From the following Table of F, you then find $F_{.999(2,18)}$ = ______________.

Table 9 F's for 5%, 1%,& 0.1% Levels

n_2 \ n_1	1	2	3
17	4.45 8.40 15.72	3.59 6.11 10.66	3.20 5.18 8.73
18	4.41 8.28 15.38	3.55 6.01 10.39	3.16 5.09 8.49

$\frac{15.24}{0.68} = 22.41$

---

(129) The observed F-ratio 22.41 exceeds the critical value $F_{.999(2,18)} = 10.39$. Therefore, the __________ is rejected at the ______% level. This means that there is significant ___________ existing among the three treatments. To conclude the analysis, let us collect the results in the Table of ANOVA as follows:

Table 10 Results of Analysis (III)

Source of Variation	Sum of Squares	Degree of Freedom	Variance Estimate	F-ratio
Treatments	30.47	2	15.24	32.41***
Blocks	331.50	9	36.83	
Residual	12.20	18	0.68	
Total	374.17	29		

2

18

10.39

XI 3-1 Randomized Block Design vs. Completely Randomized Design

(130) The randomized block design we have just discussed is more sensitive than the completely randomized design. This advantage can readily be seen by assuming the ten blocks in the foregoing example as ten observations, that is, taking the experiment as as a ______________________ design rather than a randomized block design and carry out the ANOVA. Table 8-b (refer back to Frame 117) now becomes:

Table 11 X, X^2, $\sum X$, & $\sum X^2$ (III-a)

Observation	Treat. 1		Treat. 2		Treat. 3		Total	
	X	X^2	X	X^2	X	X^3	X	X^2
1	6	36	5	25	3	9		
2	5	25	4	16	3	9		
3	5	25	4	16	1	1		
4	4	16	3	9	0	0		
5	2	4	1	1	-1	1		
6	2	4	-1	1	-2	4		
7	0	0	-1	1	-2	4		
8	-2	4	-2	4	-3	9		
9	-4	16	-3	9	-4	16		
10	-6	36	-5	25	-7	49		
Total	12	166	5	107	-12	102	5	375

In a completely randomized design, there are no ________________. All subjects are assigned to the three treatments completely at random.

null hypothesis

0.1

difference

---

(131) To test the null hypothesis, that assumes that there is ____________________ among the three treatments, we partition the total variation into two independent components: (1) the ________________ variation and (2) the ________________ variation. For each source of variation we then compute the sum of squares and determine its associated number of degree of freedom. Two variance estimates (or mean squares) are computed and from which the observed F-ratio will be obtained. Finally, by consulting the tables of F for critical values, we may or may not ____________ the null hypothesis.

completely randomized

blocks

(132) First of all, let us compute the sums of squares: SS_{total} = ________ - ________ = ________________ $SS_{between}$ = ________ - ________ = ________________ SS_{within} = ________ - ________ = ________________	no difference between-treatment (or within-treatment) within-treatment (or between-treatment) reject
(133) Next, let us determine the corresponding numbers of degrees of freedom: df_{total} = ________ - ________ = ________________ $df_{between}$ = ________ - ________ = ________________ df_{within} = ________ - ________ = ________________	$\sum X^2 - \frac{(\sum X)^2}{N} = 375 - \frac{(5)^2}{30} = 374.17$ $\frac{1}{n}[(T_1)^2+(T_2)^2+(T_3)^2] - \frac{(\sum X)^2}{N} = \frac{1}{10}[(12)^2+(15)^2+(-12)^2] - \frac{(5)^2}{30} = 30.47$ $SS_{total} - SS_{between} = 374.17 - 30.47 = 343.70$
(134) We can now obtain two variance estimates or ________: $s^2_{between}$ = ________________ s^2_{within} = ________________	$N-1 = 30-1 = 29$ $K-1 = 3-1 = 2$ $df_{total} - df_{between} = 29-2 = 27$

(135) In this case the ________________ variance estimate is considered to be the error variance estimate. Thus, we obtain the observed F-ratio

F_{ob} = ________________________

mean squares

$$\frac{S^2_{between}}{df_{between}} = \frac{30.47}{2} = 15.24$$

$$\frac{S^2_{within}}{df_{within}} = \frac{342.70}{27} = 12.73$$

---

(136) By consulting the tables of F, we find $F_{.95(2,27)} = 3.35$, $F_{.99(2,27)} = 5.49$, and $F_{.999(2,27)} = 9.02$. Since $F_{ob} < F_{.95(2,27)}$ the null hypothesis is ________. That is, there is ______ ____________________ among the three treatment means.

within-treatment

$$\frac{S^2_{between}}{S^2_{within}} = \frac{15.24}{12.73} = 1.197$$

---

(137) We may conclude the analysis by presenting the results in the following table of ANOVA:

Table 12 Results of Analysis (III-a)

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F-ratio
Between T.	30.47	2	15.24	1.197
Within T.	343.70	27	12.73	
Total	374.17	29		

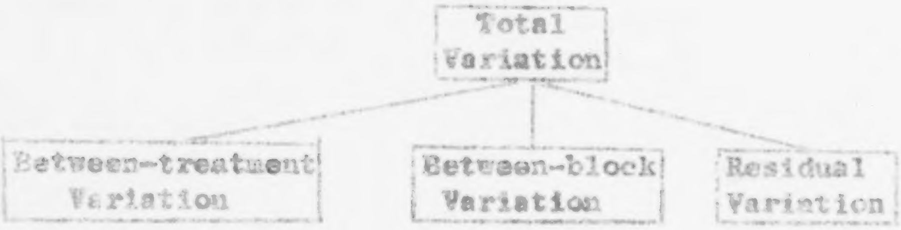
Notice that the observed F-ratio is not marked with ________________. As you may recall with the same data for the randomised block design we found highly significant result (refer back to Frame 129). This should convince you of the sensitivity of the randomised block design over the ____________________ design in the test of significance.

not rejected

no significant difference

(138) Summary In the randomized block design in which each experimental unit (plot or cell) contains only one individual subject (this is termed as non-replicated design), the total variation is partitioned into three ________________ components:

Figure 4 Partitioning of Total Variation - RBD (nonreplicated)



asterisk (s)

completely randomized

---

(139) The total sum of squares is partitioned into (1) the between-treatment sum of squares, (2) the between-block sum of squares, and (3) the residual sum of squares. That is,

$$SS_{total} = SS_{treat} + SS_{block} + SS_{residual}$$

The total sum of squares is obtained by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

Where X^2 = ________________ of each score squared

Σ = ________________ of all the scores

N = ________________________________

independent

---

(140) The between-treatment sum of squares is obtained by the formula

$$SS_{treat} = \frac{1}{n}\left[(T_1)^2 + (T_2)^2 + \ldots + (T_k)^2\right] - \frac{(\Sigma X)^2}{N}$$

Where n = number of ____________

T_k = kth treatment ____________

$\frac{(\Sigma X)^2}{N}$ = ________________ term

sum (or grand total)

sum (or grand total)

total number of cases or subjects

(141) The between-block sum of squares is obtained by the formula

$$SS_{block} = \frac{1}{k}\left[(B_1)^2 + (B_2)^2 + \dots + (B_n)^2\right] - \frac{(\Sigma X)^2}{N}$$

Where k = number of ________________

B_n = nth block ________________

blocks

total

correction

---

(142) Since we know that

$$SS_{total} = SS_{treat} + SS_{block} + SS_{residual}$$

we can obtain the residual sum of squares by

$SS_{residual}$ = ____________________

treatments

total

---

(143) The total number of degrees is partitioned into (1) the between-treatment degrees of freedom, (2) the between-block degrees of freedom, and (3) the residual degrees of freedom.

That is,

$$df_{total} = df_{treat} + df_{block} + df_{residual}$$

Let N = total number of cases (or subjects)
k = number of treatments
n = number of blocks

Then, the total degrees of freedom is obtained by the formula

df_{total} = ______________ - ____________

$SS_{total} - SS_{treat} - SS_{block}$

(144) The between-treatment degrees of freedom is obtained by the formula

$df_{treat} =$ ______________ - ______________

And the between-block degrees of freedom is determined by

$df_{block} =$ ______________ - ______________

Thus, the residual degrees of freedom may be obtained by

$df_{residual} =$ ____________________________

N-1

---

(145) Since the blocks are grouped on the basis of some background variable which is not a factor, we would compute two variance estimates (or mean squares): (1) the between-treatment variance estimate and (2) the ______________ variance estimate. The variance estimate (or mean square) is obtained by dividing the ______________ by its corresponding ______________. Thus,

$$s^2_{between} = \frac{SS_{between}}{df_{between}}$$

$$s^2_{residual} = \frac{SS_{residual}}{df_{residual}}$$

k-1

n-1

$df_{total} - df_{treat} - df_{block}$
$= (N-1) - (k-1) - (n-1)$
$= N-k-n+1$

---

(146) The observed F-ratio is then obtained by dividing the ______________ variance estimate by the ______________ variance estimate.

That is,

$$F_{ob} = \frac{s^2_{treat}}{s^2_{residual}}$$

The residual variance estimate is considered as the ______________ variance estimate here.

residual

sum of squares

number of degrees of freedom

(147) Finally, the observed F-ratio is compared with the critical F values. To do this, you have to look up the tables of F for the critical values. The value of df_{treat} gives you the number of _______________ and the value of $df_{residual}$ gives you the number of _______________ of the tables of F in which the critical value is to be found.	between-treatment residual error
(148) If the observed F-ratio _______________ the critical F value, the null hypothesis is rejected. You will, then, conclude by saying that there is __________________ among the treatment means.	column row

II 4. Randomized Treatment Design on Same Subjects (Single-Factor Design with Replicated Measures on Same Subjects)

(149) So far, we have discussed the single-factor designs with non-replicated (or non-repeated) measures in which (1) each experimental unit (plot or cell) contains only one individual subject and (2) the subjects assigned to the experiment are all different from cell to cell. If we have more than one subject in each experimental unit (plot or cell), or the same subject is assigned to more than one experimental unit (plot or cell), the experiment is no longer one with ____________________ measures.

exceeds

significant
difference

---

(150) In other words, we would have an experiment with replicated measures if (1) each experimental unit (plot or cell) contains ____________ individual subject or (2) the same subject is assigned to ____________ cell (or treatment condition).

non-replicated

(151) Replication, in experiment, can be used (1) to provide a measure of experimental error, (2) to balance out the effects of uncontrolled variables, and (3) to spot gross errors in the measurements. Thus, with replication we may be able to improve the __________________ of our experiments.

more than one

more than one

---

(152) In educational and psychological research the individual differences often constitute a source of relatively large variability. To provide an estimate of this source of variability which is separated from the treatment effects and experimental error, replication may be used. Consider now a single-factor experiment in which ten subjects have been drawn from a population at random and each subject has been observed under three different treatment conditions with these treatments assigned at a random order. This is a ______________ single-factor design which may be called a randomized treatment design on same subjects. Suppose we observed the following results:

Table 13-a Observed Data (IV)

Subject	Treat. 1	Treat. 2	Treat. 3
1	17	26	30
2	12	18	26
3	10	14	24
4	11	19	18
5	14	14	21
6	18	21	34
7	21	30	38
8	19	28	28
9	16	25	20
10	20	24	21

sensitivity (or power)

(153) Let us code the data by subtracting from each score in Table 13-a a constant of ________ and rearrange the table, we have

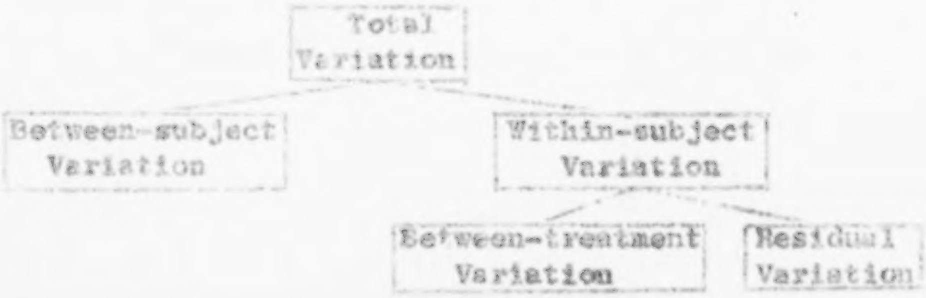
replicated
(or repeated)

Table 13-b X, X^2, ΣX, & ΣX^2 (IV)

Subject	Treat. 1		Treat. 2		Treat. 3		Total	
	X	X^2	X	X^2	X	X^2	X	X^2
1	-3	9	6	36	10	100	13	145
2	-8	64	-2	4	6	36	-4	104
3	-10	100	-6	36	4	16	-12	152
4	-9	81	-1	1	-2	4	-12	86
5	-6	36	-6	36	1	1	-11	73
6	-2	4	1	1	14	196	13	201
7	1	1	10	100	18	324	29	425
8	-1	1	8	64	8	64	15	129
9	-4	16	5	25	0	0	1	41
10	0	0	4	16	1	1	5	17
Sum	-42	312	19	319	60	742	37	1373

(154) Now, the sources of variation should be determined. That is, the total variation is to be ________________. In this case, the total variation may be partitioned into two statistically independent components: (1) between-subject variation and (2) within-subject variation. The within-subject variation is, then, further subdivided into two parts: (1) between-treatment variation and (2) residual variation. Schematically the partitioning of the total variation can be shown as follows: Figure 5 Partioning of Total Variation - RTD

20



Now, our Table of ANOVA looks like this:

Table 14-a Source of Variation

Source of variation	SS	df	s^2	F
Between subj.				
Within subj.				
Treatments				
Residual				
Total				

(155) The sums of squares for all sources of variation are to be computed next. To compute the total sum of squares, the following formula is used.

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{nk} \; . \; . \; . \; (XI\text{-}28)$$

Where n = number of subjects, k = number of treatments. Substituting the values of

$\sum X^2$, $\sum X$, and N from Table 13-b (Frame 153) in the formula, we have

$$SS_{total} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

partitioned

---

(156) The between-subject sum of squares is computed by the formula

$$SS_{bet\text{-}sub} = \frac{1}{k}[(S_1)^2 + (S_2)^2 + \ldots + (S_n)^2] - \frac{(\sum X)^2}{nk} \ldots (XI\text{-}29)$$

Where n = number of subjects

k = number of treatments

s_n = nth subject total

Substituting the values from Table 13-b (Frame 153), we have

$$SS_{bet\text{-}sub} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$1373 - \frac{(37)^2}{30}$$

$$= 1327.37$$

(157) From Figure 5 (Frame 154), we know that the total sum of squares is partitioned into (1) the between-subject sum of squares and (2) the within-subject sum of squares. That is,

$$SS_{total} = SS_{bet\text{-}sub} + SS_{within\text{-}sub} \ldots \text{(II-30)}$$

Thus, substituting the values of SS_{total} and $SS_{bet\text{-}sub}$ in the formula, we have the within-sub. sum of squares

$SS_{within\text{-}sub}$ = ______________________

$\frac{1}{3}[(13)^2 + (-4)^2 + (-12)^2 +$

$(-11)^2 + (13)^2 + (20)^2 +$

$(15)^2 + (1)^2 + (5)^2] -$

$\frac{(37)^2}{30}$

$= 572.70$

---

(158) Now, the within-subject sum of squares should be subdivided into (1) the between-treat. sum of squares and (2) the residual sum of squares. That is,

$$SS_{within\text{-}sub} = SS_{treat} + SS_{residual} \ldots \text{(II-31)}$$

The between-treatment sum of squares can be computed by the formula

$$SS_{treat} = \frac{1}{n}[(T_1)^2 + (T_2)^2 + \ldots + (T_k)^2] - \frac{(\Sigma X)^2}{nk} \ldots \ldots \text{(II-32)}$$

where n = number of subjects
k = number of treatments
T_k = kth treatment total

Substituting the values from Table 13 -b (Frame 153) in the formula, we have

SS_{treat} = ______________ - ______________

= ______________________

$SS_{total} - SS_{bet\text{-}subj}$

$= 1327.37 - 572.70$

$= 754.67$

(159) Substituting the values of $SS_{within\ sub}$ and SS_{treat} in Formula II-31 shown in the preceding Frame, we obtain the residual sum of squares.

SS_{resid} = ____________ - ____________

____

Our Table of ANOVA now looks like this:

Table 14-b Sum of Squares

Source of Variation	Sum of Squares	df.	s^2	F
Between T	572.70			
Within T	754.67			
Treat.	526.87			
Resid.	227.80			
Total	1,327.37			

or

$$\frac{1}{10}\left[(-42)^2 + (19)^2 + (60)^2\right] - \frac{(37)^2}{30} = 526.87$$

---

(160) Next, we should determine the numbers of degree of freedom corresponding to these sums of squares. The total number of degrees is ________ less than the ________ number of cases (or scores) in the experiment. That is,

$df_{total} = nk - 1$ . . . . [II-33]

Substituting the values of n and k in the formula, we have

df_{total} = ____________________

$SS_{within\ sub} - SS_{treat}$
$= 754.67 - 526.87$
$= 227.80$

(161) The between-subject degrees of freedom is one less than the number of ____________ in the experiment. That is,

$$df_{bet-sub} = n-1 \quad \text{........(II-34)}$$

Substituting the value of n in the formula, we have

$$df_{bet-sub} = \_\_\_\_\_\_\_\_\_\_\_\_$$

one

total

(10)(3) - 1 = 29

---

(162) The total number of degrees is partitioned into (1) the between-subject degrees of freedom and (2) the within-subject degree of freedom (refer back to Figure 5, Frame 154)
That is,

$$df_{total} = df_{bet-sub} + df_{within-sub} \quad \text{(II-35)}$$

Substituting the values of df_{total} and $df_{bet-sub}$ in the formula, we obtain the within-subject degrees of freedom

$$df_{within-sub} = \_\_\_\_\_\_\_\_\_\_\_\_$$

subjects

10-1 = 9

(163) Or, the within-subject degrees of freedom may be obtained through the reasoning like this. Within each individual subject K scores are compared against their mean score. This means that K-1 degrees of freedom exist for that particular subject. Since there are n such subjects in the experiment, the number of degrees of freedom for the within-subject sum of squares should be equal to n(K-1). That is,

$$df_{within\text{-}sub} = n(K-1) \ldots\ldots\ldots (II\text{-}36)$$

Substituting the values of n and K in the formula, we then have

$df_{within\text{-}sub} =$ ____________________

$df_{total} - df_{bet\text{-}sub}$

$= 29 - 9$

$= 20$

(164) The within-subject degrees of freedom should further be subdivided into (1) the between-treatment degrees of freedom and (2) the residual degrees of freedom (refer back to Figure 5, Frame 154). That is,

$$df_{within\text{-}sub} = df_{treat} + df_{resid} \ldots\ldots (II\text{-}37)$$

The between-treatment degrees of freedom can be obtained by the formula

$$df_{treat} = K-1 \ldots\ldots\ldots (II\text{-}38)$$

Substituting the value of K in the formula, we have

$df_{treat} =$ ____________________

10 (3-1) = 20

(165) We may determine the residual degrees of freedom by substituting the values of $df_{within-sub}$ and df_{treat} in Formula II - 37 Thus,

df_{resid} = ____________________

Our Table of ANOVA now looks like this:

Table 14-c Degree of Freedom

Source of variation	Sum of squares	Degree of freedom	S^2	F
Between S	572.70	9		
Within S	754.67	20		
Treat.	526.87	2		
Resid.	227.80	18		
Total	1327.37	28		

3 - 1 =

---

(166). We are now ready to compute the variance estimates. Since our primary interest in this single-factor experiment is in the effects of the three treatments, we shall compute two variance estimates (or mean squares) One is based on the between-treatment variation, and the other is based on the residual variation which is considered as the ____________ variation. The between-treatment variance estimate is computed by the formula

$$S^2_{treat} = \frac{SS_{treat}}{df_{treat}} \text{ ....(II-39)}$$

Substituing the values of SS_{treat} and df_{treat} (consult Table of ANOVA shown in the preceding frame), we have

s^2_{treat} = ____________________

$df_{within-sub} - df_{treat}$

= 20-2

= 18

(167) The residual variance estimate, or the residual __________________, is computed by the formula

$$s^2_{resid} = \frac{SS_{resid}}{df_{resid}} \quad \ldots\ldots\ldots \quad (XI\text{-}40)$$

Substituting the values of SS_{resid} and df_{resid} in the formula, we have

$$s^2_{resid} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

The Table of ANOVA now becomes:

Table 14-d Variance Estimate

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F
Between S	572.70	9		
Within S	754.67	20		
Treat.	526.87	2	263.43	
Resid.	227.80	18	12.66	
Total	1327.37	29		

error

$$\frac{526.87}{2} = 263.43$$

---

(168) The observed F-ratio is obtained by testing the between-treatment variance estimate against the __________________ variance estimate. That is,

$$F_{ob} = \frac{s^2_{treat}}{s^2_{resid}} \quad \ldots\ldots\ldots \quad (XI\text{-}41)$$

Substituting the values of s^2_{treat} and s^2_{resid} in the formula, we have

$$F_{ob} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

mean square

$$\frac{227.80}{18} = 12.66$$

(169) The numerator of Formula II-41, that is, the between-treatment variance estimate, has 2 degrees of freedom and the denominator of the formula, that is, the residual variance estimate, has 18 degrees of freedom. Therefore, the critical F values should be found in Column ____________ and Row ____________ of the Tables of F.

residual

$$\frac{263.45}{12.66} = 20.81$$

---

(170) Portion of the Tables of F is given below

Table 15 F's for 5 %, 1 %, and 0.1 % levels

n_1 / n_2	1	2	3
17	4.45 8.40 15.72	3.59 6.11 10.66	3.20 5.18 8.73
18	4.41 8.28 15.38	3.55 6.01 10.39	3.16 5.09 8.49
19	4.38 8.18 15.08	3.52 5.93 10.16	3.13 5.01 8.28

From this, we have critical F values:

$F_{.95(2,18)} =$ ____________ (at 5 % level),

$F_{.99(2,18)} =$ ____________ (at 1 % level),

$F_{.999(2,18)} =$ ____________ (at 0.1 % level)

2

18

(171) The observed F-ratio is 20.81 which exceeds ______________ Thus, the null hypothesis that assumes that there is no significant difference among the three treatments is ______________ beyond the ______________ % level. To conclude the analysis, let us collect the results in the following. To show the level of significance $F_{ob} = 20.81$ should be marked with ______________

Table 14-s F-Ratio

Source of variation	Sum of squares	Degree of Freedom	Variance estimate	F-ratio
Between S	572.70	9		
Within S	754.67	20		
Treat.	526.87	2	263.45	20.81^{***}
Resid.	227.80	18	12.66	
Total	1327.37	29		

3.55

6.01

0.39

---

(172) Summary The replicated single-factor design in which each subject is assigned to a set of treatments (these treatments constitute the factor) may be termed as the ______________ design.

$F_{.999(2,18)} = 10.39$

rejected

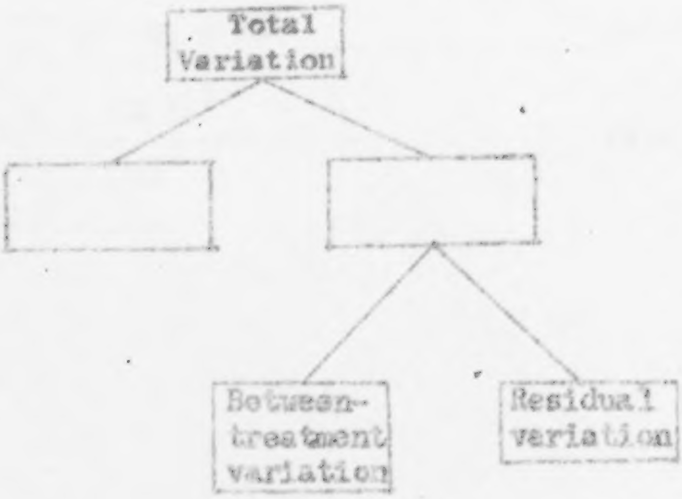
0.1

three asterisks (***)

(173) The total variation of the randomized treatment design is partioned into (1) the between-subject variation and (2) the within-subject variation. The ______________ variation is further partitioned into (3) the between-treatment variation and (4) the residual variation as shown below (Fill in blank boxes):

randomized treatment

Figure 5 Partitioning of Total Variation - RTD



---

(174) The total sum of squares is obtained by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{nk}$$

Where $\sum X^2$ = ______________ of each score squared

$\sum X$ = ______________ of all the scores

n = number of ______________

k = number of ______________

within-subject

Bet-sub. variation

within-sub. variation

(175) The between-subject sum of squares is computed by the formula

$$SS_{bet\text{-}sub} = \frac{1}{K}\left[(S_1)^2 + (S_2)^2 + \cdots \cdots\cdots + (S_n)^2\right] - \frac{(\Sigma X)^2}{nk}$$

where k = number of __________

n = number of __________

S_n = nth __________ total

$\frac{(\Sigma X)^2}{nK}$ = __________ term

grand total (or sum)

grand total (or sum)

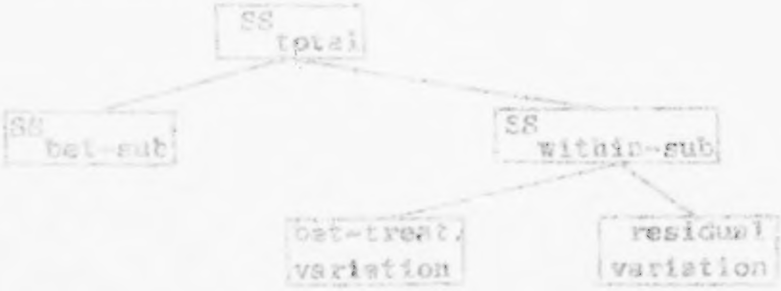
subjects

treatments

(176) Since $SS_{total} = SS_{bet-sub} + SS_{within-sub}$ the within-subject sum of squares can be obtained by

$SS_{within-sub} =$ ______________ - ______________

Figure 6 Partitioning of Total Sum of Squares - RTD



treatments

subjects

subject

correction

(177) The within-subject sum of squares is subdivided into the between-treatment sum of squares and the residual sum of squares. The between-treatment sum of squares is computed by the formula

$$SS_{treat} = \frac{1}{n}\left[(T_1)^2 + (T_2)^2 + \ldots\ldots + (T_K)^2\right] - \frac{(\Sigma X)^2}{nK}$$

Where n = ______________________

K = ______________________

$T_K =$ Kth ______________________ total

$SS_{total} - SS_{bet-sub}$

(178) Since $SS_{within\text{-}sub} = SS_{treat} + SS_{resid}$ the residual sum of squares may be obtained by

SS_{resid} = ________ - ________

number of subjects

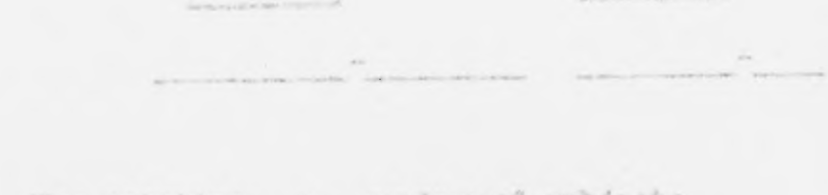
number of treatments

treatment

---

(179) The total number of degrees of freedom is partitioned as follows. Fill in the blanks.

Figure 7 Partitioning of Total Degrees of Freedom - RTD



Use notation: n = number of subjects
K = number of treatments

$SS_{within\text{-}sub}$

SS_{treat}

(130) This is a single-factor design. That is, the treatments constitute the ______________. We need to determine the between-treatment variance estimate, or the between-treatment __________, to test against the error variance estimate. The between-treatment variance estimate is obtained by the formula

$$s^2_{\text{treat}} = \frac{SS_{\text{treat}}}{df_{\text{treat}}}$$

$df_{\text{total}} = nK - 1$

$df_{\text{bet-sub}} = n - 1$

$df_{\text{within-sub}} = df_{\text{total}} - df_{\text{bet-sub}}$

$= (nK - 1) - (n - 1) = nK - n$

$= n(K - 1)$

$df_{\text{treat}} = K - 1$

$df_{\text{resid}} = df_{\text{within-sub}} - df_{\text{treat}}$

$= n(K - 1) - (K - 1)$

$= (n - 1)(K - 1)$

(181) The residual variance estimate is considered as the ______________ variance estimate in this case. It is computed by the formula $s^2_{\text{resid}} =$ ______________________	factor mean square
(183) The observed F-ratio is then obtained by testing the ______________ variance estimate against the ______________ variance estimate That is, $F_{ob} = \dfrac{S^2_{\text{treat}}}{S^2_{\text{resid}}}$	error $\dfrac{SS_{\text{resid}}}{df_{\text{resid}}}$

(183) The critical F values are found in the Tables of F. The number of degrees of freedom for S^2_{treat}, that is, df_{treat}, gives the number of __________; the number of degrees of freedom for S^2_{resid}, that is, df_{resid}, gives the number of __________ of the Tables of __________ in which the critical values are to be found.	between-treatment residual
(184) If the observed F-ratio __________ the critical value the __________ is rejected. That is, significant __________ exists among the treatments.	column row F

XI 5. Latin Square Design

(135) One of the configurations of block design is the Latin square design which represents a single-factor experiment with restricted randomization with respect to row and column effects (to be discussed in the following) in its classic setting. Consider a case in which the effects of K different fertilizers are to be tested. We may take a block of land and have it divided into plots (or experimental units) of equal areas with K rows and K columns. The K different treatments (fertilizers) are then randomly assigned to the plots in such a way that each treatment occurs only once in each row and only once in each column. This is a K × K Latin square. Assuming that the treatment effects do not interact with the row and column effects, the design constitutes a ______________ factor (or classification) experiment.

exceeds

null hypothesis

difference

(186) Conventionally, the treatments are denoted by the letters A, B, C, D, E, etc. Thus,

B C A
A B C
C A B

is a ______________ Latin square, because each of the three treatments A, B, and C appears ______________ in each row and __________ in each column. But

B C A
A A B
C B C

is not a Latin square because ________
____
____

single

---

(187) By definition, the standard form of a Latin square is that square in which the letters in the first row and the letters in the first column are in alphabetical order. For example,

A B C
B C A
C A B

is the standard form for 3x3 Latin squares. There are four different standard forms for 4x4 Latin squares. One of the standard forms is as follows:

________________.

In most of the tables of Latin squares, only the standard forms are given.

3x3

only once

only once

treatment A appears twice in the second row and treat C appears twice in the third row

(188) To obtain a Latin square for use in an experiment the following procedures should be followed. First of all, one of the standard forms of appropriate dimension, which may be obtained from the tables of Latin squares such as the Fisher-Yates tables, should be selected at random. For instance, for an experiment in which four treatments are involved, one of the standard _________________ Latin squares is to be selected at random.	Any one of the following: A B C D A B C D B A D C B C D A C D B A C D A B D C A B D A B C A B C D A B C D B D A C B A D C C A D B C D A B D C B A D C B A
(189) Suppose that we have randomly selected the following Latin square from the tables: A B C D B C D A C D A B D A B C The next step will be the selection of random sequence of digits 1 through K (the number of treatments) from the Tables of Random Numbers. For the 4x4 Latin square we would select 1, 2, 3, and 4 at _________ from the Tables, since K = _____________ in this case.	4x4

(190) Suppose that we have obtained a random sequence of 2-3-1-4 from the Tables of Random Numbers. The columns of the standard Latin square selected are then rearranged in accordance with this random sequence. That is, Column 2 of the original Latin square becomes Column 1 of the new Latin square; Column 3 becomes Column __________; Column 1 becomes Column __________; and Column 4 becomes Column __________. That is,

```
A B C D     B C A D
B C D A     C D B A
        -->
C D A B     D A C B
D A B C     A B D C
```

random

4

---

(191) Another random sequence of digits 1 through K (the number of treatments), in this case 1 through 4, is to be obtained from the Tables of Random Numbers. Suppose that we have a random sequence of 3-2-1-4 this time. Rows 3, 2, 1, and 4 of the previous Latin square we obtained in Frame 190 become Rows 1, 2, 3, 4 of a still new Latin square which will be used for the projected experiment.
That is, we have the final Latin square by rearranging the previous one (show the final Latin square to the right of arrow below):

```
B C A D
C D B A
        ------>
D A C B
A B D C          ____________________
```

2

3

4

(192) With this Latin square, let us consider an experiment in which the effects of four tests (or treatments) A, B, C, and D are evaluated. Let the four rows represent four individual subjects, and the four columns stand for the order of testing. Let us consider this as a single-factor experiment. That is, the ________________ between the rows and column effects is assumed to be non-existence.

D A C B
C D B A
B C A D
A B D C

(193) Suppose that we have observed the following data:

Table 16-a Observed Data (V)

	Order 1	Order 2	Order 3	Order 4
Subject 1	D (20)	A (35)	C (25)	B (40)
Subject 2	C (25)	D (15)	B (30)	A (30)
Subject 3	B (25)	C (20)	A (25)	D (25)
Subject 4	A (25)	B (35)	D (30)	C (15)

The effects of four ______________ A, B, C, and D can, now, be analyzed with the effects of individual ______________ of the four subjects and the ______________ of tests balanced out.

interaction

---

(194) As usual, we may code the observed data by subtracting from each score a ______________ whose magnitude is to be as near the ______________ of the observed scores as possible.

tests (or treatments)

differences

order

(195) Let us subtract 25 from each score in Table 16 and rearrange the table, we have

constant

Table 16-b X, X^2, $\sum X$, & $\sum X^2$ (V)

mean

	Order 1	Order 2	Order 3	Order 4	Subj. Total
Sub 1	(D) X X^2 -5 25	(A) X X^2 10 100	(C) X X^2 0 0	(B) X X^2 15 225	X X^2 20 350
Sub 2	(C) X X^2 0 0	(D) X X^2 -10 100	(B) X X^2 5 25	(A) X X^2 5 25	X X^2 0 150
Sub 3	(B) X X^2 0 0	(C) X X^2 -5 25	(A) X X^2 0 0	(D) X X^2 0 0	X X^2 -5 25
Sub 4	(A) X X^2 0 0	(B) X X^2 10 100	(D) X X^2 -5 25	(C) X X^2 -10 100	X X^2 -5 225
Order Total	X X^2 -5 25	X X^2 5 325	X X^2 0 50	X X^2 10 350	10 750
Treat. Total	(A) X X^2 15 125	(B) X X^2 30 350	(C) X X^2 -15 125	(D) X X^2 -20 150	

Notice that the treatment totals are obtained by summing up all the A scores, B scores, and so on.

From this table, we obtain

the correction term= $\frac{(\sum X)^2}{K \cdot K}$ = __________

This is an important quantity for the computation of the ____________________.

(constant) mean

(196) In the single-factor Latin square design the total variation is partitioned into four components: (1) the between-treatment variation, (2) the between-order variation, (3) the between-subject variation, and (4) the residual variation. In this case, the residual variation is considered to be the variation due to the ____________________ error.

$$\frac{(10)^2}{4 \times 4} = 6.25$$

sums of squares

---

(197) The total sum of squares is computed by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{K \times K} \quad \ldots\ldots (XI\text{-}42)$$

Where $\sum X^2$ = grand total of each score squared
$\sum X$ = grand total of all the scores
K = number of treatments

Substituting these values from Table 16-b in the formula, we have

SS_{total} = ________________________

experimental

(198) The between-treatment sum of squares is obtained by the formula

$$SS_{treat} = \frac{1}{k}\left[(T_1)^2 + (T_2)^2 + \ldots\ldots + (T_k)^2\right] - \frac{(\Sigma X)^2}{K \times K} \quad \ldots\ldots(II-43)$$

Where T_k = Kth treatment total

substituting the necessary values from Table 16-b (Frame 195) in the formula, we have

SS_{treat} = ________ - ________

= ________ - ________

$$750 - \frac{(10)^2}{16}$$

$= 743.75$

---

(199) The between-order sum of squares is computed by the formula

$$SS_{order} = \frac{1}{k}\left[(O_1)^2 + (O_2)^2 + \ldots\ldots + (O_k)^2\right] - \frac{(\Sigma X)^2}{K \times K} \quad \ldots\ldots(II-44)$$

Where O_k = Kth Order total
Substituting the needed values from Table 16-b (Frame 195) in the formula, we have

SS_{order} = ________ - ________

= ________ - ________

$$\frac{1}{4}\left[(15)^2 + (30)^2 + (-15)^2 + (-20)^2\right] - \frac{(10)^2}{16}$$

$= 431.25$

(200) The between-subject sum of squares is obtained by the formula

$$SS_{sub} = \frac{1}{k}[(S_1)^2 + (S_2)^2 + \ldots + (S_k)^2] - \frac{(\Sigma X)^2}{k \times k} \ldots\ldots(II-45)$$

Where S_k = kth subject total substituting the necessary values (from Table 15-b, Frame 195) in the formula, we have

SS_{sub} = ________ - ________

= ________

$$\frac{1}{4}[(-5)^2+(5)^2+(0)^2+(10)^2] - \frac{(10)^2}{16}$$

$= 31.25$

---

(201) As mentioned in Frame 196, we know that

$$SS_{total} = SS_{treat} + SS_{order} + SS_{sub} + SS_{resid} \ldots\ldots(II-46)$$

Substituting the values of SS_{total}, SS_{treat}, SS_{order}, and SS_{sub},

we have the residual sum of squares

SS_{resid} = ________ - ________

________ - ________

= ________

$$\frac{1}{4}[(20)^2+(0)^2+(-5)^2+(-5)^2] - \frac{(10)^2}{16}$$

$= 106.25$

(202) The total number of degrees of freedom is partitioned into (1) the between-treatment df, (2) the between-order df, (3) the bet.-sub. df, and (4) the residual df. That is,

$$df_{total} = df_{treat} + df_{order} + df_{sub} + df_{resid} \quad \ldots\ldots\ldots (II-47)$$

The total number of degrees of freedom is obtained by the formula

$$df_{total} = N - 1$$

Where N = total number of scores

= K x K in this case
substituting the value in the formula we have

df_{total} = ____________________

$SS_{total} - SS_{treat} - SS_{order}$

$- SS_{sub}$

= 743 75-431 25-32 25

-106 25

= 175.00

(203) Since we have the same numbers of treatments, test order, and subjects in the Latin square design, the numbers of degrees of freedom corresponding to SS_{treat}, SS_{order}, and SS_{sub} are the ____________.
That is

$$df_{treat} = df_{order} = df_{sub} = K-1 \quad \ldots\ldots (II-48)$$

Where k = number of treatments (or subjects, or test order)
Substituting the value in the formula we have

$df_{treat} = df_{order} = df_{sub} =$ ____________

(4)(4) - 1 = 15

(304) The total variation has been partitioned into four components. Therefore, we shall have four variance estimates, or four ______________
The variance estimate is obtained by dividing the ______________ by its corresponding ______________
That is,

$$s^2 = \frac{SS}{df} \quad \ldots\ldots\ldots\ldots\ldots (II-49)$$

Thus, s^2_{treat} = ______________

s^2_{order} = ______________

s^2_{sub} = ______________

s^2_{resid} = ______________

same

4-1 = 3

(205) From the previous frames, we have $SS_{treat} = 431.25$, $SS_{order} = 31.25$ $SS_{sub} = 106.25$, $SS_{resid} = 175.00$, $df_{treat} = df_{order} = df_{sub} = 3$, and $df_{resid} = 6$

Substituting these values in the equations shown above, we have

$s^2_{treat} = \underline{\qquad\qquad\qquad}$

$s^2_{order} = \underline{\qquad\qquad\qquad}$

$s^2_{sub} = \underline{\qquad\qquad\qquad}$

$s^2_{resid} = \underline{\qquad\qquad\qquad}$

mean squares

sum of squares

degrees of freedom

$$\frac{SS_{treat}}{df_{treat}}$$

$$\frac{SS_{order}}{df_{order}}$$

$$\frac{SS_{sub}}{df_{sub}}$$

$$\frac{SS_{resid}}{df_{resid}}$$

(206) Our primary concern is the effects of the four treatments. And since the individual subjects and the order of test are not the experimental factors, we would test the between-treatment variance estimate against the error variance estimate which happens to be the residual variance estimate in this case. Therefore,

$$F_{ob} = \frac{s^2_{treat}}{s^2_{resid}} \quad ....(XI\text{-}50)$$

Substituting the values of s^2_{treat} and s^2_{resid}, we have the observed F-ratio

F_{ob} = ________________________________

$\frac{431.25}{3} = 143.75$

$\frac{31.25}{3} = 10.42$.

$\frac{106.25}{3} = 35.42$

$\frac{175.00}{6} = 29.17$

(307) From the Tables of F we obtain the critical F Values as follows:

$F_{.95\ (3,6)} = 4.76$

$F_{.99\ (3,6)} = 9.78$

Since $F_{ob} = 4.93 > F_{.95(3,6)} = 4.76$

$F_{ob} = 4.93 < F_{.99(3,6)} = 9.78$

$\frac{143.75}{29.17} = 4.93$

The null hypothesis that assumes that there is no difference among the
____________________ is ________________
at ____________________ % level.

The results of the analysis are shown below.

Table 17 Results of Analysis (V)

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F-ratio
Between T	431.25	3	143.75	4.93^{*}
Between O	31.25	3	10.42	
Between S	106.25	3	35.42	
Residual	175.00	6	29.17	
Total	743.75	15		

$^{*}F_{.95(3,6)} = 4.76$

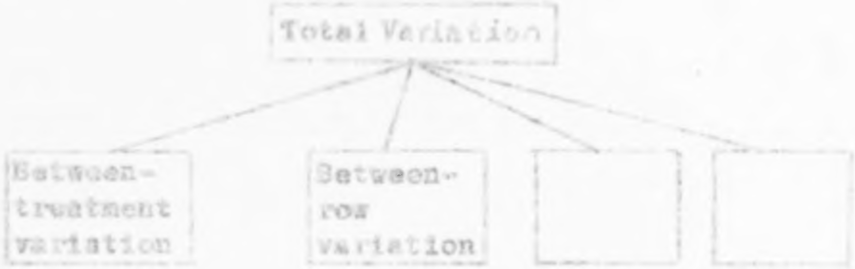
(208) Summary In its classic setting the Latin square design represents a single-factor experiment.
A k x k Latin square has k columns, k rows, and ______________ treatments which constitute the experimental factor.

treatments
(four treatments)

rejected

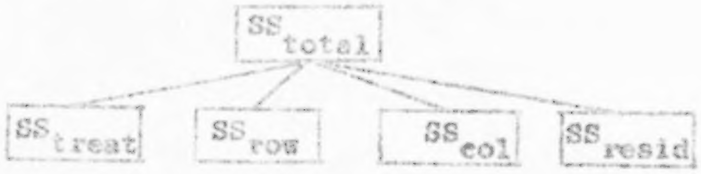
---

(209) For a single-factor k x k Latin square experiment, the total variation is partitioned into four components: (1) the between-treatment variation, (2) the between-row variation, (3) the ______________ variation, and (4) the ______________ variation.



k

(210) The total sum of squares is also partitioned into four components:



The total sum of squares is computed by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{k \times k}$$

Where $\sum X^2$ = ________ of all the squared scores

$\sum X$ = ________ of all the scores

k = number of ____________

Between-column

Residual

---

(211) The between-treatment sum of squares is obtained by the formula

$$SS_{treat} = \frac{1}{k}\left[(T_1)^2 + (T_2)^2 + \ldots\ldots + (T_k)^2\right] - \frac{(\sum X)^2}{k \times k}$$

Where k = ____________ of treatments

T_k = kth ________________

$\frac{(\sum X)^2}{k \times k}$ = ________________ Term

sum (or grand total)

sum (or grand total)

treatments (or rows or columns)

(212) The between-row sum of squares is computed by the formula

$$SS_{row} = \frac{1}{k}\left[(R_1)^2 + (R_2)^2 + \ldots\ldots\ldots\ldots + (R_k)^2\right] - \frac{(\Sigma X)^2}{k \times k}$$

Where k = number of __________

R_k^2 kth __________

$\frac{(\Sigma X)^2}{k \times k}$ = __________

the number

treatment total

the correction

---

(213) The between-column sum of squares is computed by the formula

$$SS_{col} = \frac{1}{k}\left[(C_1)^2 + (C_2)^2 + \ldots\ldots\ldots\ldots + (C_k)^2\right] - \frac{(\Sigma X)^2}{k \times k}$$

Where k = __________ of rows

C_k = __________ column __________

columns (or rows or treatments)

row total

correction term

(214) We know that

$$SS_{total} = SS_{treat} + SS_{row} + SS_{col} + SS_{resid}$$

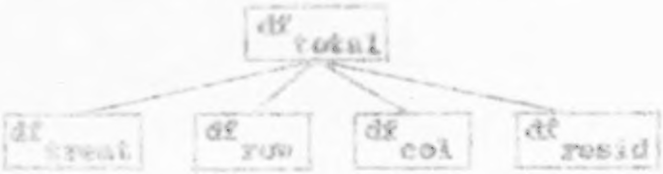
Therefore, we can obtain the residual sum of sum of squares as follows:

SS_{resid} = ________ - ________ - ________ - ________

number

kth total

(215) The total number of degrees of freedom is partitioned into four components:



The total degrees of freedom can be obtained by the formula

$$df_{total} = N - 1$$

Where N = ________ of cases

For nonreplicated design N = k x k
Where k = number of ________________

$SS_{total} - SS_{treat}$

$-SS_{row} - SS_{col}$

(216) In the $k \times k$ Latin square we have k treatments, k rows, and k columns. Thus,

$$df_{treat} = df_{row} = df_{col} = \_\_\_\_\_\_\_\_ - 1$$

Since we know that

$$df_{total} = df_{treat} + df_{row} + df_{col} + df_{resid}$$

The residual degrees of freedom can be computed by

$$df_{resid} = \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$$

$$\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$$

total number

rows (or columns)

(217) Four variance estimates, or ________ ________, may be obtained as follows:

$$s^2_{treat} = \frac{SS_{treat}}{df_{treat}}$$

$$s^2_{row} = \frac{SS_{row}}{df_{row}}$$

$$\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

k

$df_{total} - df_{treat} -$

$df_{row} - df_{col}$

(218) This is a single-factor experiment (the k treatments constitute the factor) we assume that there is no interaction between the rows and the columns. In other words, we are interested in testing the between-treatment variance estimate against the error variance estimate. We would then have an observed

________________ ratio:

$$F_{ob} = \frac{S^2_{treat}}{S^2_{resid}}$$

The residual variance estimate, s^2_{resid}

is considered as the ____________________ variance estimate here.

$$s^2_{col} = \frac{SS_{col}}{df_{col}}$$

$$s^2_{resid} = \frac{SS_{resid}}{df_{resid}}$$

(219) The critical F values will have to be obtained next. These values are to be found in the Tables of F with the value of df_{treat} giving the number of ________________

and the value of df_{resid} giving the number of

________________ of the Table.

F

error

(230) Finally, the observed F-ratio is compared with the critical F values. If the critical values exceed the observed F-ratio, the null hypothesis is ____________________. That is, there is ____________________ difference among the k treatment means.

column

row

III MULTI-FACTOR EXPERIMENTS

(221) So far we have dealt with the single-factor (or classification) experimental designs in which data were classified on the basis of a single experimental variable (factor, classification, or criterion). Let us now consider the cases in which two or more experimental variables (factors, classifications, or criteria) are involved in the experiment. These may be called the ________________ experimental designs.

not rejected

no significance

---

(222) Factorial experiments: One of very important designs of the multifactor experiments is the so-called factorial experiment. This is an extremely useful design in the analysis of variance. The factorial experiments are concerned with simultaneous influence of ____________ or more independent experimental variables (classifications or criteria), usually called ________________.

multi-factor

(or classification)

(223) In the factorial experiment, we control several factors and investigate their combined effects at each of two or more levels. The term levels is used to indicate the number of ways in which a factor is varied. Thus, a factor which is varied in two ways would be said to have ______________ levels. A factor which has three levels is varied (or classified into) in ____________ ways (or categories).

two

factors

---

(224) The dimensions of a factorial design are expressed by the number of factors and the number of levels of each factor. For instance, a factorial design with two factors, one having two levels and the other having three levels is called a 2x3 factorial design. In a 2x3x4x5 factorial experiment, there are ____________ factors, with each factor having ______________________ levels, respectively.

A factorial experiment with n factors, each at two levels is called a ______________ factorial experiment.

two

three

(230) In a 2 x 2 factorial experiment, there are four different treatment combinations (or cells) possible as shown in Table 18 in which Factor A has levels a_1 and a_2, and Factor B has levels b_1 and b_2. Thus, for a p x q factorial experiment,

Table 18 . 2 x 2 Factorial

Design

	a_1	a_2
b_1		
b_2		

Table 19 2x2x2 Fact.

Design

	a_1		a_2	
	b_1	b_2	b_1	b_2
c_1				
c_2				

there are ________________ different treatment combinations. In a 2 x 2 x 2 factorial experiment as shown in Table 19, there are 2 x 2 x 2 = 8 possible treatment combinations (or cells). Thus, for a p x q x r experiment, there are ________________ possible treatment combinations (or cells). Similarly, for a 2^n factorial experiment, there are ________________ different treatment combinations (or cells).

four

two, three, four and five

2^n

(226) In the analysis of factorial experiments, we speak of main effects and interactions (or interaction effects). The main effects of a given factor is the comparisons between the levels of that factor. Taking Table 19, for example, the main effects of Factor A is a measure of the extent to which the mean of all observations at level A_1 and A_2 differ from each other.

Thus, in the same example, the main effects of Factor C is the comparisons between ________________ and ________________.

$p \cdot q$

$p \cdot q \cdot r$

2^n

(227) The combined effects of different factors are termed as interactions (or interaction effects). The interaction between effects A and B is denoted by A x B. Interactions between pairs of factors are called the first order interactions; interactions between triplets of factors are termed as the second order interactions; etc. Thus, A x B denotes a ________________ order interaction between A and B. A x B x C denotes a ________________ order interaction between A, B, and C. A x B x C x D x E is a ________________ order interaction showing the combined effects of A, B, C, D, and E.

C_1

C_2

XIX. A. 2 x 2 Factorial Design Having One Observation Per Cell (Non-Replicated)

(228) Suppose that we have conducted an experiment in which two factors with each factor having two levels were observed simultaneously. This is a ______________ factorial experiment, with ______________ treatment combinations. In this experiment only one observation was made for each of the four treatment combinations (or cells). The observed data are shown below:

Table 30-a Observed Data (VI) - 2 x 2 Factorial Non-replicated

	a_1	a_2
b_1	61	64
b_2	66	75

first

second

fourth

2 x 2

(229) As usual, let us code the observed data. Substracting from each score a constant ______________, we have

4

Table 20-b Coded Form (VI)

	a_1	a_2	Total
b_1	-4	-1	-5
b_2	1	10	11
Total	-3	9	6

---

(230) The total variation can be ______________ into three components: variations due to (1) the A effects, (2) the B effect, and (3) the A x B interaction. Our Table of ANOVA will look like this

65

Table 21-a Source of Variation (VI)

Source of Variation	SS	df	s^2	F
A B A x B				
Total				

(231) With the coded data, now let us compute the sums of squares.

	a_1	a_2	Total
b_1	-4	-1	-5
b_2	1	10	11
Total	-3	9	6

The total sum of squares is computed by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} \quad \ldots\ldots\ (III\text{-}1)$$

Where $\sum X^2$ = sum of all the squared scores

$\sum X$ = sum of all the scores

N = total number of scores in the experiment

Substituting these values (from the table) in the formula, we have

$$SS_{total} = [\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_] - \_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_$$

partitioned

(332) The A effect sum of squares is obtained by the formula

$$SS_A = \frac{1}{n}[(A_1)^2+(A_2)^2] - \frac{(\Sigma X)^2}{N} \quad \ldots\ldots(III\text{-}2)$$

Where n = number of scores in each level of the A factor

A_i = the a_i level total

$\frac{(\Sigma X)^2}{N}$ = ______________ term

Substituting these values in the formula, we have

SS_A = ______________-______________

= ______________

$[(-4)^2+(1)^2+(-1)^2$
$+(10)^2] - \frac{(6)^2}{4}$

= 118-9

= 109

(333) You may use a short-cut formula to compute the A effect sum of squares. This is given by

$$SS_A = \frac{(A_1-A_2)^2}{N} \quad \ldots\ldots\ldots\ldots(III - 3)$$

	a_1	a_2	Total
b_1	-4	-1	-5
b_2	1	10	11
Total	-3	9	6

Substituting these values in the formula, we have

SS_A = ______________-______________

= ______________

correction

$\frac{1}{2}[$[illegible]$] - \frac{(6)^2}{4}$

= 45-9

= 36

(235) The B effect sum of squares is computed by the formula

$$SS_B = \frac{1}{k}\left[(B_1)^2 + (B_2)^2\right] - \frac{(\Sigma X)^2}{N} \ldots (III\text{-}4)$$

where k = number of scores in each level of the B factor
B_1 = the b_1 level total

$\frac{(\Sigma X)^2}{N}$ = ______________ term

Substituting these values in the formula, we have

SS_B ______________ - ______________
____

$\frac{(-3-9)^2}{4} = 36$

(236) Again, you may use a short-cut formula to obtained the B effect sum of squares. This is given by

$$SS_B = \frac{(B_1 - B_2)^2}{N} \ldots\ldots\ldots\ldots (III - 5)$$

	a_1	a_2	Total
b_1	-4	-1	-5
b_2	1	10	11
Total	-3	9	6

Substituting the necessary values in the formula, we have

SS_B = ______________________________

correction

$$\frac{1}{2}\left[(-5)^2 + (11)^2\right]$$

$$- \frac{(6)^2}{4} = 73 - 9$$

$= 64$

(236) We know that

$$SS_{total} = SS_A + SS_B + SS_{A \times B} \quad \ldots (III\text{-}6)$$

Therefore, substituting the values of SS_{total}, SS_A, and SS_B, we have the A x B sum of squares

$$\begin{aligned} SS_{A \times B} &= \underline{\qquad\qquad} - \underline{\qquad\qquad} \\ &= \underline{\qquad\qquad} - \underline{\qquad\qquad} \\ &= \underline{\qquad\qquad} \end{aligned}$$

In our present example (non-replicated design), the A x B sum of squares may be considered as the residual sum of squares.

$$\frac{(-5-11)^2}{4} = 64$$

(237) To obtain a first-order interaction sum of squares, $S_{A \times B}$, the following formula may be used

$$SS_{A \times B} = \frac{\left[(X_{a_1b_1}+X_{a_2b_2})-(X_{a_1b_2}+X_{a_2b_1})\right]^2}{N}$$

.... (III-7)

	a_1	a_2
b_1	-4	-1
b_2	1	10

Substituting the necessary values from the table in the formula, we have

$SS_{A \times B}$ = ____________________

$SS_{total} - SS_A - SS_B$

= 109 - 36 - 64

= 9

---

(238) Now our Table of ANOVA looks like this:

Table 21-b Sum of Squares (VI)

Source of variation	Sum of squares	df	s^2	F
A B A x B	36 64 9			
Total	109			

Before we can obtain the variance estimates (or mean square). The ____________ associated with the sums of squares must be determined.

$$\frac{[(-4+10)-(1-1)]^2}{4}$$

$$= \frac{(6)^2}{4} = 9$$

(239) The total number of degrees of freedom is partitioned into (1) the A effect degrees of freedom, (2) the B effect degrees of freedom, and (3) the A x B interaction degrees of freedom. That is,

$$df_{total} = df_A + df_B + df_{A \times B} \ldots (III-8)$$

The total number of degrees of freedom is obtained by the formula

$$df_{total} = N - 1 \ldots\ldots (III-9)$$

Where N = total number of scores. Substituting the value of N in the formula, we then have

df_{total} = ____________________

degrees of freedom (dfs)

---

(240) The A effect degrees of freedom is determined by the formula

$$df_A = p-1 \ldots\ldots\ldots (III-10)$$

Where p = number of levels of the A factor

Substituting the value of p in the formula, we have

df_A = ____________________

4 - 1 = 3

(241) The B effect degrees of freedom is computed by the formula

$$df_B = q-1 \text{ ....(III -11)}$$

Where q = number of levels of the B factor

Substituting the value of q in the formula, we then have

df_B = ____________________

2-1 = 1

---

(242) We know that

$$df_{total} = df_A + df_B + df_{A \times B}$$

Thus, the A x B interaction degrees of freedom can be obtained by substituting the values of df_{total}, df_A, and df_B in the formula

$df_{A \times B}$ = __________ - __________

- ____________________

= ____________________

Or, you may obtain it by the formula

$$df_{A \times B} = df_A \times df_B \text{ ....(III -12)}$$

Substituting the values of df_A and df_B, we have

$df_{A \times B}$ = ____________________

2 - 1 = 1

(243) The Table of ANOVA now becomes:

Table 21-C Degrees of Freedom (VI)

Source of variation	Sum of Squares	Degree of Freedom	s^2	F
A	36	1		
B	64	1		
A x B	9	1		
Total	109	3		

We may now obtain the variance estimate for each source of variation by dividing its ________________ by the corresponding ____________________.

$df_{total} - df_A - df_B$

$= 3 - 1 - 1$

$= 1$

$1 \times 1 = 1$

(244) The variance estimate is also known as the ________________ in ANOVA. It is defined as

$$s^2 = \frac{SS}{df} \quad \text{..........(III -13)}$$

Thus, substituting the values of sums of squares and numbers of degrees of freedom in the formula, we have three variance estimates as follows:

$s^2_A =$ ____________________

$s^2_B =$ ____________________

$s^2_{A \times B} =$ ____________________

sum of squares (SS)

number of degrees of freedom (df)

(245) The results are entered in the Table of ANOVA as follows:

Table 21-d Variance Estimates (VI)

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F
A	36	1	36	
B	64	1	64	
A x B	9	1	9	
Total	109	3		

The observed F-ratio can now be obtained by testing the variance estimate of our interest against the error variance estimate. That is

$$F_{ob} = \frac{s^2}{s^2_{error}} \quad \ldots\ldots\ldots \text{(XXI -14)}$$

To find the appropriate ______________ variance estimate is indeed a matter of concern in the F-test.

mean square

$$\frac{SS_A}{df_A} = \frac{36}{1}$$

$= 36$

$$\frac{SS_B}{df_B} = \frac{64}{1}$$

$= 64$

$$\frac{SS_{A \times B}}{df_{A \times B}} = \frac{9}{1}$$

$= 9$

---

(246) A discussion concerning the selection of appropriate variance estimate is now in order. We must now consider the ANOVA models and expectations of variance estimates. In a multi-factor experiment if the levels (or treatments) of each factor are selected at random from some larger populations, it is referred to as a random ANOVA model; if the levels (or treatments) are not randomly selected, it is referred to as a fixed ANOVA model; if the levels (or treatments) of some factors are randomly selected and that of others are not, it is called a mixed ANOVA model. Let us now return to our example. Suppose that we have randomly selected a_1 and a_2 for factor A and also b_1 and b_2 for factor B. We have a ______________ ANOVA model.

error

(247) Expectations of variance estimates (or components of variances) for the random ANOVA model are given by the following table.

Table 22 Expectations of Variance Estimates A and B both Random

Source	Expectations of V. E.
A	$\sigma^2 + \sigma_{ab}^2 + b\sigma_a^2$
B	$\sigma^2 + \sigma_{ab}^2 + a\sigma_b^2$
A x B	$\sigma^2 + \sigma_{ab}^2$

For each factor we assign a lower-case letter. When we use the letter as a subscript we use it to designate a source of variation. When the letter is used as a coefficient we use it to designate the number of levels of the source. In our present example we assign the letter
____________________ for the A factor
and the letter ____________________
for the B factor.

random

(248) In the table presented above,

σ^2 = error variance

σ_a^2 = variance directly attributable to A effect

σ_b^2 = variance directly attributable to ____________

σ_{ab}^2 = variance directly attributable to ____________

Notice that the coefficient b is associated with σ_a^2; the coefficient ____________ is associated with σ_b^2 in the table.

a

b

---

(249) It must be mentioned that the coefficient of a component for a given source consists of all letters not used as subscripts for that source. Thus, in our example in which two letters a and b are assigned to the two factors, the coefficient of σ_a^2 is b, since b is not used as subscript. Similarly the coefficient of σ_b^2 is ____________ and the coefficient of

σ_{ab}^2 is ____________

B effect

A x B interaction

a

(250) The expectations (or components) of the variance estimate for a given source always include the error variance, designated σ^2, and a component of variance directly attributable to that source ($b\sigma_a^2$ for A for instance) plus the other components whose subscripts contain the letter (or letters) necessary to describe the source under consideration. Thus for A, we also have ______________ since the subscripts contain a and this is the only letter necessary to describe completely the source, namely A. (refer back to Table 22, Frame 247)

a

1 (both a and b are used as subscripts)

---

(251) The appropriate error term, which is the denominator of an F-ratio, for a test of a given effect is the variance estimate whose components contain all the components of the variance estimate of the given effect except the one directly attributable to the given effect. In other words, the appropriate error term for the A effect, whose components are $\sigma^2 + \sigma_{ab}^2 + b\sigma_a^2$, should be ____________, since $b\sigma_a^2$ is the component directly attributable to the A effect. That is,

$$F_{ab} = \frac{\sigma^2 + \sigma_{ab}^2 + b\sigma_a^2}{\sigma^2 + \sigma_{ab}^2}$$

σ_{ab}^2

(252) Referring back to Table 22, Frame 247, you should find that

$\sigma^2 + \sigma_{ab}^2$ is the expectation (or components) of the A x B variance estimate. Thus, to test the A effect in our present example, which is considered as a random ANOVA model, A x B is the appropriate error term. To test the B effect whose expectation of variance estimate being

$\sigma^2 + \sigma_{ab}^2 + a\,\sigma_b^2$, the term having expectation of variance estimate of ____________________ should be used as the error term. Again, you will find ________________ to be the appropriate error term.

$\sigma^2 + \sigma_{ab}^2$

---

(253) How do we select the appropriate error term for an F test of the A effect or the B effect if we have the fixed ANOVA model? Suppose that in our example we have not randomly selected the levels a_1 and a_2 for the factor A, and the levels b_1 and b_2 for the factor B (refer back to Table 20-a, Frame 228). We now have a ____________________ ANOVA model.

$\sigma^2 + \sigma_{ab}^2$

A x B

(254) When we have fixed effects, the expectations, or ______________, of the variance estimates are different from that of the random model. Certain components are to be deleted from the expectations of the variance estimates of the random model.

The error variance, designated σ^2, and the component due directly to a fixed effect,

designated θ^2 for the fixed effect, however, are not to be deleted. For example, for the A effect, in a random model, the expectation of variance estimate is

$\sigma^2 + \sigma_{ab}'^2 + b\,\sigma_a^2$. For the fixed model,

the expectation of variance estimate of the A effect will retain σ'^2 and $b\,\sigma_a'^2$ which is directly attributable to the A effect and will be denoted by ______________.

fixed

---

(255) That leaves the component

$\sigma_{ab}'^2$ which may or may not be deleted.

To determine whether or not a component should be deleted, the procedures go like this. First the subscript letter (or letters) which is necessary to describe the source is deleted.

That is, $\sigma_{ab}'^2$ becomes $\sigma_b'^2$. Next,

delete the component if one of the remaining subscripts represents a fixed effect. Since the subscript of

$\sigma_b'^2$, __________, represents

a __________ effect, the component

σ_{ab}^2 is ______________.

components

$b\,\theta_a^2$

(256) Now, we have the expectations of variance estimates for the fixed ANOVA model as follows:

Table 2 Expectations of Variance Estimates A and B both Fixed

Source	Expectation of V. E.
A	$\sigma^2 + \qquad b\,\theta_a^2$
B	$\sigma^2 + \quad a\,\theta_b^2$
A x B	σ^2

In the table

^{2} = error variance

$b\ {}^2_a$ = variance directly attributable to ______________

$a\ {}^2_b$ = variance directly attributable to ______________

b

fixed

deleted

(257) The appropriate error term for a test of a given effect is, as we have seen, that variance estimate whose components contain all of the components of the variance estimate of the given effect ______________ the one directly attributable to the given effect. In other words, the appropriate error term for the A effect of the fixed ANOVA model shown in Table 23 is ______________ because we can obtain $F_{ob} = \frac{\sigma^2 + b \cdot \theta_a^2}{\sigma^2}$ whose denominator has the same component of the variance estimate of numerator except $b\theta_a^2$ which is directly attributable to ______________.	the A effect the B effect
(258) For the same reason, A x B is the appropriate error term to be used in testing the B effect. So in the 2 x 2 factorial experiments, the A x B effect is the ______________ against which the A and B effects may be tested whether it be a random ANOVA model or a fixed ANOVA model.	except A x B the A effect

(259) As shown in Table 21-d, Frame 245 we have $s^2_A = 36$; $s^2_B = 64$ and $s^2_{A \times B} = 9$. Thus, we shall have two F-ratios:

$$F_A = \frac{s^2_A}{s^2_{A \times B}} \quad \text{..........(XXI -15)}$$

$$F_B = \frac{s^2_B}{s^2_{A \times B}} \quad \text{..........(XXI -16)}$$

The A x B variance estimate, designated $s^2_{A \times B}$, is considered as the ____________ variance here. Substituting the necessary values in the equations, we have

F_A = ________________________________

F_B = ________________________________

appropriate error term

(260) From the Tables of F the following cirtical F values are obtained:

$$F_{.95(1,1)} = 161$$

$$F_{.99(1,1)} = 4052$$

Comparing with the observed F values $F_A = 4$ and $F_B = 7.11$, you find that the critical values are much larger. Therefore, the hypotheses that assume that

$\bar{X}_{a_1} = \bar{X}_{a_2}$ and $\bar{X}_{b_1} = \bar{X}_{b_2}$ are ______________.

Table 21-c F-Ratios (VI)

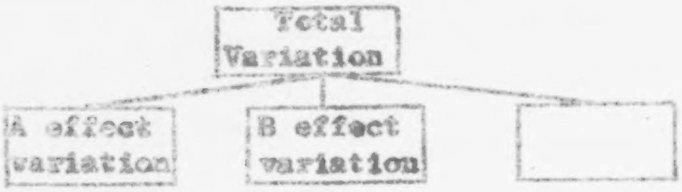
Source of variation	Sum of squares	Degree of freedom	Variance Estimate	F-ratio
A	36	1	36	4
B	64	1	54	7.11
AxB	9	1	9	
Total	109	3		

error

$\frac{36}{9} = 4$

$\frac{64}{9} = 7.11$

(261) Summary For a 2 x 3 non-replicated factorial experiment with A and B denoting the two factors the total variation is partitioned into three components: (1) the A effect variation, (2) the B effect variation, and (3) the ________________ variation.



not rejected

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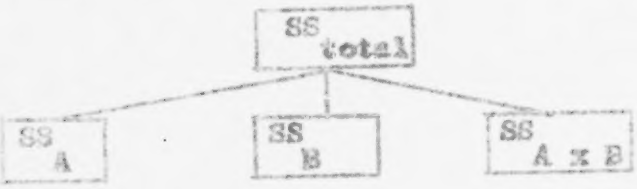
(262) The total sum of squares is, therefore, partitioned into ________________ components.



The total sum of squares is computed by the formula

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N}$$

Where $\sum X^2$ = ________________ of all the squared scores

$\sum X$ = ________________ of all the scores

N = ________________ of scores

A x B effect

(or residual)

(263) The A effect sum of squares is computed by the formula

$$SS_A = \frac{1}{n}[(A_1)^2 + (A_2)^2] - \frac{(\Sigma X)^2}{N}$$

Where n = number of scores in each ________ of the A factor

A_1 = the a_1 level ________

$\frac{(\Sigma X)^2}{N}$ = ________ term

	a_1	a_2	Total
b_1	X_{11}	X_{12}	B_1
b_2	X_{21}	X_{22}	B_2
Total	A_1	A_2	X

three

sum (or grand total)

sum (or grand total)

total number

(264) Or you may obtain the A effect sum of squares by a short cut formula which is given by

$$SE_A = \frac{(A_1 - A_2)^2}{N}$$

This formula is in effect a **simplified** form of the formula given in Frame **263**. That is,

$$SS_A = \frac{1}{n}\left[(A_1)^2 + (A_2)^2\right] - \frac{(\sum X)^2}{N}$$

$$= \frac{2}{2n}\left[(A_1)^2 + (A_2)^2\right] - \frac{(A_1 + A_2)^2}{N}$$

$$= \frac{2(A_1)^2 + 2(A_2)^2 - (A_1 + A_2)^2}{N}$$

= ________________________

$$= \frac{(A_1)^2 + (A_2)^2 - 2(A_1)(A_2)}{N}$$

= ________________________

level

total

correction

(265) The B effect sum of squares is computed by the formula

$$SS_B = \frac{1}{K}\left[(B_1)^2 + (B_2)^2\right] - \frac{(\sum X)^2}{N}$$

Where n = number of scores in each
__________ of the __________

B_1 = the b_1 level __________

$\frac{(\sum X)^2}{N}$ = __________ term

Or you may use the following short cut formula

SS_B = __________

$$\frac{1}{N}\left\{2(A_1)^2+2(A_2)^2-\left[(A_1)^2+(A_2)^2+2(A_1)(A_2)\right]\right\}$$

$$\frac{(A_1-A_2)^2}{N}$$

(266) We know that

$$SS_{total} = SS_A + SS_B + SS_{A\times B}$$

Therefore, the A x B effect sum of squares may be obtained as follows:

$SS_{A\times B}$ = __________ - __________
- __________

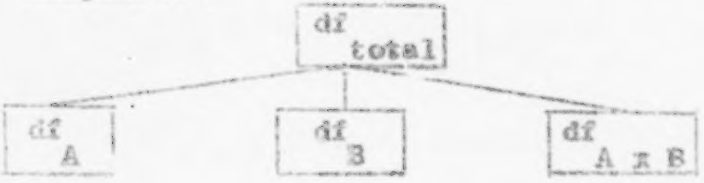
level B factor

total

correction

$$\frac{(B_1-B_2)^2}{N}$$

(267) The total number of degrees of freedom is also partitioned into three components:



As usual, the total number of degrees of freedom is obtained by the formula

df_{total} = ________ – ________

Where N = ______________ of scores

$SS_{total} - SS_A - SS_B$

---

(268) Since we have 2 x 2 factorial design, the A factor has ________ levels and the B factor has ________ levels. The degrees of freedom for the A effect is ________ less than its number of levels. Therefore,

df_A = ________ – ________

= ________________

Similarly,

df_B = ________ – ________

= ________________

N – 1

total number

(269) To obtain $df_{A \times B}$ you may utilize the relation

$$df_{total} = df_A + df_B + df_{A \times B}$$

Thus,

$$df_{A \times B} = \underline{\qquad\qquad} - \underline{\qquad\qquad} - \underline{\qquad\qquad}$$

Or, you may use the formula

$$df_{A \times B} = df_A \times df_B$$

Substituting the values of df_A and df_B we obtained in the foregoing frame, we have the A x B effect (or residual) degrees of freedom:

$$df_{A \times B} = \underline{\qquad\qquad\qquad}$$

two

two

one

2 - 1 = 1

2 - 1 = 1

---

(270) The variance estimate which is often referred to in ANOVA as the ______________ is obtained by dividing the ______________ by its corresponding number of ______________.
Therefore, three variance estimates may be computed by the following formulas:

$$s^2_A = \underline{\qquad\qquad\qquad};$$

$$s^2_B = \underline{\qquad\qquad\qquad};$$

$$s^2_{A \times B} = \underline{\qquad\qquad\qquad}$$

$df_{total} - df_A - df_B$

$(1) \cdot (1) = 1$

(271) In the nonreplicated 2 x 2 factorial experiment two F-ratios, F_A and F_B, can be obtained by testing s_A^2 and s_B^2 against ________________ if we have the random ANOVA model or fixed ANOVA model. That is, when the two levels of both factors, A and B, are selected at random or they are not randomly selected we have

$$F_A = \frac{s_A^2}{s_{A \times B}^2}$$

$$F_B = \frac{s_B^2}{s_{A \times B}^2}$$

Here the A x B variance estimate, designated $s_{A \times B}^2$, is considered as the ________________ variance. Notice that we have not considered the mixed ANOVA model. We shall take up this specific model later in the program.

mean square

sum of squares

degrees of freedom

$$\frac{SS_A}{df_A}, \frac{SS_B}{df_B},$$

$$\frac{SS_{A \times B}}{df_{A \times B}}$$

---

(272) The critical F values will be obtained from the Tables of F. Finally, the critical F values are compared with the observed F value. If the observed F-ratio exceeds the critical F value the null hypothesis is ________________.

$s_{A \times B}^2$

error

rejected

XXI. 2. 2 x 2 x 2 Factorial Design Having n Observations Per Cell (Replicated Measures)

(273) Now let us consider a 2 x 2 x 2 factorial experiment. In this particular type of design there are ______________ factors with each factor having ______________ different levels (or treatments). Let A, B, and C denote the three factors; a_1, a_2; b_1 b_2; c_1, and c_2 represent the levels of A, B, and C respectively. Supose that we have observed data as shown in the following table.

Table 24-a Observed Data (VXI) – 2 x 2 x 2 Factorial Replicated

	a_1		a_2	
	c_1	c_2	c_1	c_2
b_1	7 8 9	7 8 8	7 7 6	8 9 10
b_2	2 3 3	3 4 5	5 6 6	3 4 5

Notice that we have ______________ observations per cell (or treatment combination).

(274) In ANOVA the first step has to do with the determination of the ______________ of variation. The total variation of the experiment is to be ______________ into independent components of variation. Since we have three factors, there are a total of ${}_3C_1 + {}_3C_2 + {}_3C_3 = 7$ possible combinations of the factors. These include three (${}_3C_1$) main factors, namely, A, B, and C; three (${}_3C_2$) first-order interactions, A x B, A x C, and B x C; one (${}_3C_3$) second-order interaction.

three

two

three

---

(275) Thus, the total variation can be partitioned into these seven sources plus the residual variation as shown in the following table.

Table 25-a Source of Variation (VII)

Source of variation	SS	df	s^2	F
A				
B				
C				
A x B				
A x C				
B x C				
A x B x C				
Residual				
Total				

It must be mentioned that we have residual variation for the replicated factorial experiment in which each cell (or treatment combination) contains more than ______________ observation. In other words, there will be no residual variation for the ______________ factorial design.

sources (or components)

partitioned

(276) The next step is to compute the sum of squares. Let us, first, compute the total sum of squares with the following formula:

$$SS_{total} = \sum X^2 - \frac{(\sum X)^2}{N} \text{ ....(XII-17)}$$

Where X = raw score
N = total number of cases

$$\frac{(\sum X)^2}{N} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Substituting the necessary values from Table 24, Frame 273 in the formula we have

$$SS_{total} = (7)^2+(8)^2+.......+(4)^2+(5)^2 - \frac{(7+8+........+4+5)^2}{24}$$

$$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

one

non-replicated

(377) There are three methods with which we may compute the three main effect sums of squares. Let us take the A effect sum of squares first.

Table 24-b Subtable 1

A_1	A_2	Total
67	78	145

A conventional formula is given by

$$SS_A = \frac{1}{n}\left[(A_1)^2 + (A_2)^2\right] - \frac{(\sum X)^2}{N}$$

........(XXX-18)

Where n = number of scores in each level of the A factor

A_i = the a_i level total

Substituting the values given in Subtable 1 in the formula, we have

SS_A = ____________________ - ____________________

= ________________________________

correction term

$$997 - \frac{(145)^2}{24}$$

= 120.958

(378) You may use a short cut formula which is actually a simplified form of the one we have just used. This short cut formula is given by

$$SS_A = \frac{(A_1 - A_2)^2}{N} \quad \ldots\ldots\ldots\ldots\text{(XXX-19)}$$

Substituting the values of A_1, A_2, and N in the formula, we have

SS_A = ____________________

$$\frac{1}{12}\left[(67)^2 + (78)^2\right]$$

$$- \frac{(145)^2}{24} = 5.042$$

---

(379) Let us derive Formula XXX-19 from Formula XXX-18

$$SS_A = \frac{1}{n}\left[(A_1)^2 + (A_2)^2\right] - \frac{(\sum X)^2}{N}$$

$$= \frac{2\left[(A_1)^2 + (A_2)^2\right]}{2n} - \frac{(A_1 + A_2)^2}{N}$$

$$= \frac{2\left[(A_1)^2 + (A_2)^2\right] - (A_1 + A_2)^2}{N}$$

= ____________________

= ____________________

$$= \frac{(A_1 - A_2)^2}{N}$$

$$\frac{(67-78)^2}{24} = 5.042$$

(280) A third method has something to do with so-called "orthogonal comparisons". With this method you are suppose to construct a table like this:

Table 26-a Orthogonal Comparisons-
2 x 2 x 2 Factorial

Main Effect and Interaction	Treatment Combinations							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A								
B								
C								
AxB								
AxC								
BxC								
AxBxC								

Notice that since this is a 2 x 2 x 2 factorial, we have __________ treatment combinations (or cells). Number in parenthesis for each treatment combination represents the sum of scores in that cell (refer back to Table 24-a, Frame 273).

$$\frac{1}{N}\Big[\underline{2(A_1)^2+2(A_2)^2}$$

$$-(A_1)^2+(A_2)^2$$

$$+2A_1A_2\Big]$$

$$\frac{(A_1)^2+(A_2)^2-2A_1A_2}{N}$$

(281) To compute SS_A, you must first enter the signs "+" and "-" for Row A under the eight treatment combinations in Table 26-a shown above. Take $a_1\ b_1\ c_1$, finding another combination which has $b_1\ c_1$ to form a pair such that the difference between the two will give you $(a_2-a_1)\ b_1\ c_1$. You thus find the combination $a_2b_1c_1$ to pair with $a_1b_1c_1$. Enter "+" for $a_2b_1c_1$ and "-" for $a_1b_1c_1$. Next, take $a_1b_2c_1$. The combination which can be paired with this one is $a_2b_2c_1$. Assign "+" to $a_2b_2c_1$, and "-" to $a_1b_2c_1$. Take $a_1b_1c_2$. This should be paired with ________________. Assign "+" to ________________ and "-" to ____________.

8

The results are shown below:

Table 26-b Orthogonal Comparisons

Effect	Treatment Combination							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	-	-	-	-	+	+	+	+

(282) The A effect sum of squares is then computed by the formula

$$SS_A = \frac{1}{N}(-a_1b_1c_1 - a_1b_2c_1 - a_1b_1c_2 - a_1b_2c_2 + a_2b_1c_1 + a_2b_2c_1 + a_2b_1c_2 + a_2b_2c_2)^2 \quad \ldots\ldots (III\text{-}20)$$

Where N = total number of scores
Substituting the values given in parenthesis in Table 26-b, we have

SS_A = ______________________________

$a_2b_1c_2$

$a_2b_1c_2$

$a_1b_1c_2$

---

(283) Similarly, we may compute SS_B with this method. But first let us use the conventional formula:

$$SS_B = \frac{1}{n}\left[(B_1)^2 + (B_2)^2\right] - \frac{(\Sigma X)^2}{N}$$

$$\ldots\ldots\ldots (III\text{-}21)$$

Where n = number of scores in each level of the B factor

B_1 = the b_1 level total

Table 24-c Subtable 2

B_1	B_2	Total
96	49	145

Substituting the values of n, B_1, B_2, ΣX, and N in the formula, we have

SS_B = ______________ - ______________

= ______________________________

$$\frac{1}{24}(-24-8-23-12+32+17+27+12)^2$$

= 5.042

(284) We may of course utilize the short cut formula which is

$$SS_B = \frac{(B_1-B_2)^2}{N} \quad \text{........(XXI-22)}$$

Where B_1 = the a_1 level total

N = total number of scores substituting the values of B_1, B_2, and N, we have

SS_B = ________________________

$$\frac{1}{12}\left[(96)^2+(49)^2\right] - \frac{(145)^2}{24} = 5.042$$

---

(285) With "orthogonal comparisons" SS_B can be obtained as follows. The "+" and "−" signs are to be marked for the eight treatment combinations for Row B of the Table of Orthogonal Comparisons.

Table 26-c Orthogonal Comparisons

Effect	Treatment Combination							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	−	−	−	−	+	+	+	+
B	−	+	−	+	−	+	−	+

For the first treatment combination $a_1 b_1c_1$ another combination is to be found and paired. This combination must have a_1c_1. Thus, combination $a_1b_2c_1$ should be the one. The difference of the paired combinations gives us $(b_2-b_1)a_1c_1$. Assign "+" to $a_1b_2c_1$ and "−" to $a_1b_1c_1$. Next the combination $a_1b_1c_2$ should be paired with combination ________________; $a_2b_1c_1$ should be paired with ________________; $a_2b_1c_2$ should be paired with ______________. The results are shown in the Table.

$$\frac{(96-49)^2}{24} = 92.042$$

(286) The B effect sum of squres is then obtained from the formula

$$SS_B = \frac{1}{N}(-a_1b_1c_1 + a_1b_2c_1 - a_1b_1c_2 + a_1b_2c_2 - a_2b_1c_1 + a_2b_2c_1 - a_2b_1c_2 + a_2b_2c_2)^2 \;\ldots\ldots(III\text{-}23)$$

Where N = total number of scores
Substituting all the necessary values, we have

SS_B = ______________________________

$a_1b_2c_2$

$a_2b_2c_1$

$a_2b_2c_2$

---

(287) With the conventional formula, the C effect sum of squares is computed as follows:

$$SS_C = \frac{1}{n}\left[(C_1)^2 + (C_2)^2\right] - \frac{(\Sigma X)^2}{N} \;\ldots\ldots(III\text{-}24)$$

Where n = number of scores in each level of the C effect
C_1 = the c_1 level total

Table 24-d   Subtable 3

C_1	C_2	Total
71	74	145

Substituting the values of n, c_1, c_2, ΣX, and N in the formula, we have

SS_C = ________________ − ______________

= ______________________________

$$\frac{1}{24}(-24+8-23+12-32+17-27+12)^2$$

$= 92.042$

(288) You may use a short cut formula which is given by

$$SS_c = \frac{(C1 - C_2)^2}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (III-25)$$

Where C_1 = the C_1 level total

N = total number of scores

Substituting the values of C_1, C_2, and N we have

SS_c = ______________________

$\frac{1}{12}$ $(71)^2 + (74)^2$

$-\frac{(145)^2}{24}$

$= 0.375$

---

(289) Let us now compute SS_c with "orthogonal comparisons". The "+" and "-" signs are entered for Row C in the Table of Orthogonal Comparisons as follows:

Table 26-d Orthogonal Comparisons

Effect	Treatment combination							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	-	-	-	-	+	+	+	+
P	-	+	-	+	-	+	-	+
C	-	-	+	+,	-	-	+	+

The paired combinations are

$a_1b_1c_1$ vs. __________

__________ vs __________

__________ vs. __________

__________ vs. __________

$\frac{(71-74)^2}{24} = 0.375$

(290) Then the C effect sum of squares is obtained by

$$SS_c = \frac{1}{N}(-a_1b_1c_1 - a_1b_2c_1 + a_1b_1c_2 + a_1b_2c_2 - a_2b_1c_1 - a_2b_2c_1 + a_2b_1c_2 + a_2b_2c_2)^2 \quad .....(III\text{-}26)$$

Where N = total number of scores. Substituting all the necessary values, we have

SS_c = ____________________

$a_1b_1c_2$

$a_1b_2c_1$ vs. $a_1b_2c_2$

$a_2b_1c_1$ vs. $a_2b_1c_2$

$a_2b_2c_1$ vs. $a_2b_2c_2$

(291) We now proceed to compute the sums of squares for the three ________________ - order interactions A x B, A x C, and B x C. as we have seen, to accomplish this, we may use one of three methods. We can use (1) a conventional formula, (2) a short cut formula, or (3) ____________________ comparisons.

$$\frac{1}{24}(-24-8+23+12-22-17+27+12)^2$$

$= 0.375$

(292) Let us take the A x B interaction first. The A x B sum of squares may be computed by the conventional formula which is given by

$$SS_{A \times B} = \frac{1}{n}(a_1b_1^2 + a_1b_2^2 + a_2b_1^2 + a_2b_2^2) - \frac{(\Sigma X)^2}{N} - SS_A - SS_B \;\ldots\ldots\text{(XIII-27)}$$

Where n = number of scores in each cell
ΣX = grand total of all the scores
N = total number of cases
a_1b_1 = the a_1b_1 cell total

Table 24-c Subtable 4

	a_1	a_2
b_1	47	49
b_2	20	29

$n = N/4 = 8$

$\Sigma X = 145$

Substituting all the necessary values in the formula, we have

$SS_{A \times B}$ = ______________ - ______________

= - ______________ - ______________

= ______________

first

orthogonal

(293) This A x B sum of squares can be computed by the short cut formula

$$SS_{A \times B} = \frac{1}{N}\left[(a_1b_1+a_2b_2)-(a_1b_2+a_2b_1)\right]^2 \text{..........(III-28)}$$

Where N = total number of scores
a_1b_1 = the a_1b_1 cell total

Table 24-c Subtable 4

	a_1	a_2
b_1	47	49
b_2	20	29

Substituting all the necessary values in the formula, we have

$SS_{A \times B}$ = ____________________

$$\frac{1}{6}\left[(47)^2+(20)^2+(49)^2+(29)^2\right] - \frac{(145)^2}{24} - 5.042 - 92.042 = 2.042$$

(294) With the "orthogonal comparison" the AxB sum of squares is obtained as follows. Enter the "+" and "-" signs for Row AxB in the Table of Orthogonal Comparisons.

Table 26-e Orthogonal Comparisions

Effect	Treatment Combination							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	-	-	-	-	+	+	+	+
B	-	+	-	+	-	+	-	+
C	-	-	+	+	-	-	+	+
AxB	+	-	+	-	-	+	-	+

The rules for determining the sign for a specific treatment combination in Row AxB are these:

(1) assign "+" if the entries of Row A and Row B have the same sign;

(2) assign "-" if the entries of Row A and Row B have different signs.

The entry of $a_1b_1c_1$ for Row AxB is marked with a "+" sign because the corresponding entries of Row A and Row B have ______________ signs. The entry of $a_1b_2c_1$ for Row AxB is marked with a "-" sign because the corresponding entries of Row A and Row B have ______________ signs.

$$\frac{[(47+29)-(20+49)]^2}{24}$$

$=2.042$

(295) Thus, we can now formulate the equation

$$SS_{A \times B} = \frac{1}{N}(a_1b_1c_1 - a_2b_2c_1 + a_1b_1c_2 + a_1b_2c_2 - a_2b_1c_1 + a_2b_2c_1 - a_2b_1c_2 + a_2b_2c_2)^2 \ldots\ldots\ldots(III\text{-}29)$$

Where N = total number of scores. Substituting all the necessary values in the formula, we have

$SS_{A \times B}$ = ____________________

the same ("-")

different

---

(296) Next, take the A x C interaction.

Table 24-f Subtable 5

	a_1	a_2
c_1	32	39
c_2	35	39

n = N/4 = 6

$\Sigma X = 145$

The A x C sum of squares can be computed by the formula

$$SS_{A \times C} = \frac{1}{n}({a_1c_1}^2 + {a_1c_2}^2 + {a_2c_1}^2 + {a_2c_2}^2) - \frac{(\Sigma X)^2}{N} - SS_A - SS_C \ldots\ldots(III\text{-}30)$$

Where n = number of scores in each cell

a_1c_1 = the a_1c_1 cell total

ΣX = sum of all the scores

N = total number of scores

Substituting all the necessary values in the formula, we have

$SS_{A \times C}$ = ______________ - ______________

= - ______________ - ______________

= ____________________

$$\frac{1}{24}(24-8+23-12-22+17-27+12)^2$$

$= 2.042$

(297) A short cut formula with which you may obtain $SS_{A \times C}$ is given by

$$SS_{A \times C} = \frac{1}{N} [(a_1c_1 + a_2c_2) - (a_1c_2 + a_2c_1)]^2 \quad \ldots\ldots \text{(XXI-31)}$$

Where N = total number of scores

a_1c_1 = the a_1c_1 cell total

Table 24-Y Subtable 5

	a_1	a_2
c_1	32	39
c_2	35	39

Substituting all the necessary values in the formula, we have

$SS_{A \times C}$ = ____________________

$$\frac{1}{6}[(32)^2+(35)^2+(39)^2+(39)^2] - \frac{(145)^2}{24} - 5.042 - 0.375 = 0.375$$

(238) To compute the A x C sum of squares with "Orthogonal comparisons" you first enter the "+" and "−" signs for Row A x C in the Table of Orthogonal comparisons as shown below.

$\frac{1}{24}\left[(32+39)-(35+39)\right]^2 = 0.375$

Table 26-f Orthogonal comparisons

Treatment Combination

Effect	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (23)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	−	−	−	−	+	+	+	+
B	−	+	−	+	−	+	−	+
C	−	−	+	+	−	−	+	+
A x B	+	−	+	−	−	+	−	+
A x C	+	+	−	−	−	−	+	+

The entry of $a_2b_2c_1$ for Row A x C is marked with a "−" sign because the corresponding entries of Row ____________ and Row ____________ have different signs; the entry of $a_2b_1c_2$ for Row A x C is marked with a "+" sign because the corresponding entries of Row A and Row B have the __________ sign

A

C

Same (+)

---

(239) We then formulate the equation

$$SS_{A \times C} = \frac{1}{N}(a_1b_1c_1 + a_1b_2c_1 - a_1b_1c_2 - a_1b_2c_2 - a_2b_1c_1 - a_2b_2c_1 + a_2b_1c_2 + a_2b_2c_2)^2 \quad \ldots\ldots (III\text{-}32)$$

Where H = ______________ of scores. Substituting all the necessary values in the formula, we have

$SS_{A \times C}$ = __________________

(303) Then take another first-order interaction B x C.

Table 24-g Subtable 6

	c_1	c_2
b_1	46	50
b_2	25	24

$n = N/4 = 6$

$\Sigma X = 145$

The B x C sum of squares may be computed by the conventional formula

$$SS_{B \times C} = \frac{1}{n} (b_1c_1^2 + b_2c_1^2 + b_1c_2^2 + b_2c_2^2) - \frac{(\Sigma X)^2}{N} - SS_B - SS_C \quad . \; . \; . \; . \; (III\text{-}33)$$

Where n = number of scores in each ________________

$b_1c_1 = b_1c_1$ cell ______________

ΣX = sum of all the scores

N = total number of scores

Substituting all the necessary values in the formula, we have

$SS_{B \times C}$ = ______________ - __________ + ___________

- ______________ - ______________

= _____________________________

total number

$$\frac{1}{24} (24+8-23-12-22-17+27+12)^2$$

$= 0.375$

(20_) Of course $SS_{B \times C}$ can be obtained by using the short cut formula

$$SS_{B \times C} = \frac{1}{N}\left[(b_1c_1 + b_2c_2) - (b_2c_1 + b_1c_2)\right]^2$$

. . . . . . (XXI-34)

Where N = __________________ of scores

b_1c_1 = the b_1c_1 __________________

Table 24-g Subtable 6

	c_1	c_2
b_1	46	50
b_2	25	24

Substituting all the necessary values in the formula, we have

$SS_{B \times C}$ = ________________________

cell

total

$$\frac{1}{6}\left[(46)^2 + (25)^2 + (50)^2 + (24)^2\right] - \frac{(145)^2}{24}$$

$= 92.043 - 0.375$

$= 1.042$

(30$_7$) With "orthogonal comparisons" the B x C sum of squares is obtained as follows. Enter the "+" and "-" signs for Row B x C in the Table of Orthogonal Comparisons.

Table 26-g Orthogonal Comparisons

i	Treatment Combination							
Effect	$a_1b_1c_1$	$a_1b_2c_1$	$a_1b_1c_3$	$a_1b_2c_2$	$a_2b_1c_1$	$a_2b_2c_1$	$a_2b_1c_2$	$a_2b_2c_2$
	(24)	(8)	(23)	(12)	(22)	(17)	(27)	(12)
A	-	-	-	-	+	+	+	+
B	-	+	-	+	-	+	-	+
C	-	-	+	+	-	-	+	+
A x B	+	-	+	-	-	+	-	+
A x C	+	+	-	-	-	-	+	+
B x C	+	-	-	+	+	-	-	+

The entry of $a_1b_1c_1$ for Row B x C is marked with a "+" sign because the corresponding entries of Row ____________ and Row ____________ have the same "-" sign. The entry of $a_1b_2c_1$ for Row B x C is marked with a "-" sign because the corresponding entries of Row B and Row C have ______________ signs.

B

C

different

(30$_8$) Now we have the equation formulated on the basis of the results shown in Table 26-g.

$$SS_{B \times C} = \frac{1}{N}(a_1b_1c_1 - a_1b_2c_1 - a_1b_1c_2 + a_1b_2c_2 + a_2b_1c_1 - a_2b_2c_1 - a_2b_1c_2 + a_2b_2c_2)^2 \quad \ldots\ldots (III\text{-}35)$$

Where N = ______________ of scores substituting all the necessary values in the formula, we have

$SS_{B \times C} =$ ____________________

total number

total

$\frac{1}{24}[(46+24)-(25+50)]^2$
$=1.042$

(80½) Let us proceed to the next source of variation, A x B x C, the only ________ - order interaction in our present program.

Table 24-h Subtable 7

	a_1		a_2	
	c_1	c_2	c_1	c_2
b_1	24	23	22	27
b_2	8	12	17	12

N n=N/8 = 3

$\Sigma X = 15$

The A x B x C sum of squares may be computed by a conventional formula

$$SS_{A \times B \times C} = \frac{1}{n}\left(a_1b_1c_1^2 + a_1b_2c_1^2 + a_1b_1c_2^2 + a_1b_2c_2^2 + a_2b_1c_1^2 + a_2b_2c_1^2 + a_2b_1c_2^2 + a_2b_2c_2^2\right) - \frac{(\Sigma X)^2}{N} - SS_A - SS_B - SS_C - SS_{A \times B} - SS_{A \times C} - SS_{B \times C} \quad \text{. . . . . (III-36)}$$

Where n = number of scores in each cell

$a_1b_1c_1$ = the $a_1b_1c_1$ cell total

X = sum of all the scores

N = total number of scores

Substituting all the necessary values in the formula, we have

$SS_{A \times B \times C}$ = ________ - ________

- ________ - ________ - ________

- ________ - ________ - ________

= ____________________

total number

$$\frac{1}{24}(24-8-23+12+22-17-27+12)^2$$

$= 1.042$

(305) You can save the laborious computation by using a short cut formula.

Table 24-h Subtable 7

	a_1		a_2		
	c_1	c_2	c_1	c_2	
b_1	24	23	22	27	→ T_1
B_2	8	12	17	12	→ T_2

This short cut formula is given by

$$SS_{A \times B \times C} = \frac{1}{N} (T_1 - T_2)^2 \qquad \ldots\ldots\ldots \text{(III-37)}$$

Where N = total number of scores

$$T_1 = a_2b_1c_1 + a_1b_2c_2 + a_2b_2c_1 + a_2b_1c_2$$

$$T_1 = a_1b_2c_1 + a_1b_1c_2 + a_2b_1c_1 + a_2b_2c_2$$

Substituting these values in the formula, we have

$SS_{A \times B \times C} =$ ____________________

$$\frac{1}{3}\,[(24)^2+(8)^2+(23)^2$$

$$+(12)^2+(22)^2+(17)^2+(27)^2$$

$$+(12)^2] - \frac{(145)^2}{24}$$

$$-5.042-92.042-0.375$$

$$-2.042 - 0.375 - 1.042$$

$$= 9.375$$

(306) With "orthogonal comparisons" you can also save your time in computing the AxBxC sum of squares.

Table 26-h Orthogonal Comparisons

Effect	Treatment Combination							
	$a_1b_1c_1$ (24)	$a_1b_2c_1$ (8)	$a_1b_1c_2$ (23)	$a_1b_2c_2$ (12)	$a_2b_1c_1$ (22)	$a_2b_2c_1$ (17)	$a_2b_1c_2$ (27)	$a_2b_2c_2$ (12)
A	-	-	-	-	+	+	+	+
B	-	+	-	+	-	+	-	+
C	-	-	+	+	-	-	+	+
AxB	+	-	+	-	-	+	-	+
AxC	+	+	-	-	-	-	+	+
BxC	+	-	-	+	+	-	-	+
AxBxC	-	+	+	-	+	-	-	+

To assign an appropriate sign for each entry in Row AxBxC, the corresponding entries in Rows A, B, and C are investigated. If you have an odd number of "-" signs in these entries, an "-" sign is assigned. If you have an even number of "-" signs or no "-" sign, a "+" sign is assigned. Thus, the entry of $a_1b_1c_1$ for Row AxBxC is marked with a "-" sign because we have an ________ number of "-" signs in the corresponding A, B, and C entries; the entry of $a_1b_2c_1$ for Row AxBxC is marked with a "+" sign because we have an __________ number of ________ signs in the corresponding A, B, and C rows.

$$\frac{1}{24}\left[(24+12+17+27)-(8+23+22+12)\right]^2 = 9.375$$

(30") Based on this table we are now able to formulate an equation

$$SS_{A \times B \times C} = \frac{1}{N}(-a_1b_1c_1 + a_1b_2c_1 + a_1b_1c_2 - a_1b_2c_2 + a_2b_1c_1 - a_2b_2c_1 - a_2b_1c_2 + a_2b_2c_2)^2 \quad \ldots\ldots \text{(III-38)}$$

Where N = total number of scores. Substituting all the necessary values (refer to Table 26-h) in the formula, we have

$SS_{A \times B \times C} =$ ________________

odd

even

= =

(308) Finally, let us determine the residual sum of squares. Since we know that:

$$SS_{total} = SS_A + SS_B + SS_C + SS_{AxB} + SS_{AxC} + SS_{BxC} + SS_{AxBxC} + SS_{resid} \qquad \text{(III-39)}$$

Substituting:

$SS_A = 5.042$

$SS_B = 92.042$

$SS_C = 0.375$

$SS_{AxB} = 2.042$

$SS_{AxC} = 0.375$

$SS_{BxC} = 1.042$

$SS_{AxBxC} = 9.375$

$SS_{total} = 120.958$

in the formula, we have:

SS_{resid} = __________ - __________ - __________

- __________ - __________ - __________

- __________ - __________

= ______________________________

$$\frac{1}{24}(-24+8+23-12+22-17-27+12)^2$$

$$= 9.375$$

(309) Our table of ANOVA now looks like this:

Table 25-b Sums of Squares (VII)

Source of variation	Sum of squares	df	s^2	F
A	5.042			
B	92.042			
C	0.375			
AxB	2.042			
AxC	0.375			
BxC	1.042			
AxBxC	9.375			
Residual	10.665			
Total	120.958			

According to this table, we are suppesed to determine the __________ for each source of variation next.

120.958-5.042-92.042

-0.375-2.042-0.375

-1.042-9.375

= 10.665

---

(310) The total number of degrees of freedom is given by the formula

$$df_{total} = N-1 \quad . \ . \ . \ . \ (III\text{-}39)$$

Where N denotes the total number of scores in the experiment. Since there are a total of 24 scores,

df_{total} = __________ - __________

= __________

degrees of freedom

(311) Let us now consider the degrees of freedom for the three main effect A, B, and C. Since all the three factors have the same number of levels

$df_A = df_B = df_C = K-1$ . . . . (III-40)

Where K represents the number of levels. This is a 2 x 2 x 2 factorial. Thus,

$df_A = df_B = df_C =$ __________ - __________

$=$ ____________________

24-1 = 23

(312) As to the degrees of freedom for the three ____________-order interactions, A x B, A x C, and B x C,

$df_{A \times B} = df_A \times df_B$ . . . . . . (III-41)

$df_{A \times C} = df_A \times df_C$ . . . . (III-42)

$df_{B \times C} = df_B \times df_C$ . . . . (III-43)

Substituting the values of df_A, df_B, and df_C in the formulas, we have

$df_{A \times B} =$ ____________________

$df_{A \times C} =$ ____________________

$df_{B \times C} =$ ____________________

2-1 = 1

(315) The number of degrees of freedom for the only ______________ - order interaction in the experiment, A x B x C, is determined by

$$df_{A \times B \times C} = df_A \times df_B \times df_C \quad \ldots\ldots(XXI\text{-}44)$$

Substituting the values of df_A, df_B, and df_C in the formula, we have

$$df_{A \times B \times C} = \underline{\qquad\qquad\qquad}$$

first

1 x 1 = 1

1 x 1 = 1

1 x 1 = 1

---

(316) Since we know that

$$df_{total} = df_A + df_B + df_C + df_{A \times B} + df_{A \times C} + df_{B \times C}$$

$$+ df_{A \times B \times C} + df_{resid} \quad \ldots\ldots(XXI\text{-}45)$$

The residual degrees of freedom may be obtained by substituting the values of df_{total}, df_A, df_B, df_C, $df_{A \times B}$, $df_{A \times C}$, $df_{B \times C}$, and $df_{A \times B \times C}$ in the formula.

Thus, $df_{resid} = \underline{\qquad} - \underline{\qquad} - \underline{\qquad}$

$- \underline{\qquad} - \underline{\qquad} - \underline{\qquad}$

$- \underline{\qquad} - \underline{\qquad}$

$= \underline{\qquad\qquad\qquad}$

second

1 x 1 x 1 = 1

(315) Let us collect these df's in our Table of ANOVA as follows:

Table 25-c Degrees of Freedom (VII)

Source of variation	Sum of squares	Degrees of freedom	s^2	F
A	5.042	1		
B	92.042	1		
C	0.375	1		
AxB	2.042	1		
AxC	0.375	1		
BxC	1.042	1		
AxBxC	9.375	1		
Residual	10.665	16		
Total	120.958	23		

According to this table, we are now ready to compute the ______________.

23-1-1-1-1-1-1-1

= 16

---

(316) The variance estimate, also known as the mean square in ANOVA, is defined as

$$s^2 = \frac{SS}{df}$$

Substituting the values of SS's and df's, therefore, we can compute eight variance estimates as follows:

$s^2_A =$ ______________, $s^2_B =$ ______________,
$s^2_C =$ ______________, $s^2_{A \times B} =$ ______________,
$s^2_{A \times C} =$ ______________, $s^2_{B \times C} =$ ______________,
$s^2_{A \times B \times C} =$ ______________, $s^2_{resid} =$ ______________

variance estimates (or mean squares)

(317) Our Table of ANOVA now looks like this:

Table 35-d Variance Estimates (VII)

Source of variation	Sum of squares	Degree of Freedom	Variance estimate	F
A	5.042	1	5.042	
B	92.042	1	92.042	
C	0.375	1	0.375	
A x B	2.042	1	2.042	
A x C	0.375	1	0.375	
B x C	1.042	1	1.042	
A x B x C	9.375	1	9.375	
Residual	10.665	16	0.667	
Total	120.958	23		

We are about to determine the observed ______________________

$$\frac{SS_A}{df_A} = \frac{5.042}{1} = 5.042$$

$$\frac{SS_B}{df_B} = \frac{92.042}{1} = 92.042$$

$$\frac{SS_C}{df_C} = \frac{0.375}{1} = 0.375$$

$$\frac{SS_{A \times B}}{df_{A \times B}} = \frac{2.042}{1} = 2.042$$

$$\frac{SS_{A \times C}}{df_{A \times C}} = \frac{0.375}{1} = 0.375$$

$$\frac{SS_{B \times C}}{df_{B \times C}} = \frac{1.042}{1} = 1.042$$

$$\frac{SS_{A \times B \times C}}{df_{A \times B \times C}} = \frac{9.375}{1} = 9.375$$

$$\frac{SS_{resid}}{df_{resid}} = \frac{10.665}{16} = 0.667$$

(318) As we have discussed in Frames 246 through 259, before we compute the observed F ratios we must consider the type of ANOVA model we have for our experiment. Without considering this we may not be able to select the appropriate error term against which the variance estimate of the source concerned is to be tested. Suppose that we have randomly selected the levels a_1 and a_2 for the factor A; b_1 and b_2 for the factor B; c_1 and c_2 for the factor C, then we have a so called ________________ ANOVA model.

F-ratios (or values)

(319) For the random ANOVA model the expectations (or components) of variance are given as follows:

random

Table 27-a Expectations of Variance Estimates

A, B, & C All Random

Source	Expectation of variance estimate
A	$\sigma^2 + n\sigma_{abc}^2 + nb\sigma_{ac}^2 + nc\sigma_{ab}^2 + nbc\sigma_a^2$
B	$\sigma^2 + n\sigma_{abc}^2 + na\sigma_{bc}^2 + nc\sigma_{ab}^2 + nac\sigma_b^2$
C	$\sigma^2 + n\sigma_{abc}^2 + na\sigma_{bc}^2 + nb\sigma_{ac}^2 + nab\sigma_c^2$
A x B	$\sigma^2 + n\sigma_{abc}^2 + nc\sigma_{ab}^2$
A x C	$\sigma^2 + n\sigma_{abc}^2 + nb\sigma_{ac}^2$
B x C	$\sigma^2 + n\sigma_{abc}^2 + na\sigma_{bc}^2$
AxBxC	$\sigma^2 + n\sigma_{abc}^2$
Residual	σ^2

In this table,

σ^2 = error variance

$abc\,\sigma_a^2$ = variance directly attributable to A effect

$nc\,\sigma_{ab}^2$ = variance directly attributable to ______ interaction.

Notice that n denotes number of replications in each treatment combination; the lower case letters a, b, c, etc. indicate the sources of variation when used as subscripts and denote the number of levels of the corresponding sources when used as coefficient. Notice further that the coefficient of a component for a given source consists of all letters not used as subscripts for that source.

(320) The appropriate error term for (an F-test of) a given effect is the variance estimate whose components contain all of the components of the variance estimate of the given effect except that component which is directly attributable to the given effect. In other words, the appropriate error term for the A effect, whose components are $\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab} + nbc\sigma^2_{a}$, should be ______________________________________,
since

$nbc\sigma^2_{a}$ is the component directly attributable to the A effect. The observed F value then will be given by

$$F_{ab} = \frac{\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab} + nbc\sigma^2_{a}}{\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab}}$$

A x B

---

(321) Investigating Table27-a, Frame 31 , you cannot, however, find a source which has $\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab}$. That is, there is no appropriate error variance estimate in the table. This hold true for other ________________ effects B and C.

$\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} +$
$nc\sigma^2_{ab}$

(322) To test the AxB interaction whose components of variance estimate are $\sigma^2+n\sigma_{abc}^2+nc\sigma_{ab}^2$, the variance estimate having components of __________ is the appropriate error term. Referring back to Table 27-a, Frame 31, you find that the ______________ interaction has the mentioned components. Thus, the __________ interaction is the appropriate error term for the ____________ interaction. Same thing holds true for the AxC and BxC interactions

main

---

(323) Based on the same reasoning the appropriate error ter for the AxBxC interaction is the ________________.
The appropriate error variance estimates for all sources of variation in a 2x2x2 random factorial experiment are given in the following table.

Table 27-b Error Variance Estimates
A, B, &C All Random

Source	Error term for F test
A	None
B	None
C	None
AxB	AxBxC
AxC	AxBxC
BxC	AxBxC
AxBxC	Residual

$\sigma^2 + n\sigma_{abc}^2$

AxBxC

AxBxC

AxB

(326) Based on these, let us now compute four F-ratios:

$$F_{AxB} = \frac{s^2_{AxB}}{s^2_{AxBxC}} \qquad \text{- - - - - - - (III-46)}$$

$$F_{AxC} = \frac{s^2_{AxC}}{s^2_{AxBxC}} \qquad \text{- - - - - - - (III-47)}$$

$$F_{BxC} = \frac{s^2_{BxC}}{s^2_{AxBxC}} \qquad \text{- - - - - - - (III-48)}$$

$$F_{AxBxC} = \frac{s^2_{AxBxC}}{s^2_{resid}} \qquad \text{- - - - - - - (III-49)}$$

Substituting $s^2_{AxB} = 2.042$

$s^2_{AxC} = 0.375$

$s^2_{BxC} = 1.042$

$s^2_{AxBxC} = 9.375$

$s^2_{resid} = 0.667$

in the formulas, we have:

F_{AxB} = ____________________

F_{AxC} = ____________________

F_{BxC} = ____________________

F_{AxBxC} = ____________________

Residual Variation

(325) The critical F values which are to be compared with the observed F ratios

$F_{AxB} = 0.218$

$F_{AxC} = 0.040$

$F_{BxC} = 0.111$

are obtained from the Tables of F and shown below.

$F_{.95(1,1)} = 161$ - - - - - - at 5% level

$F_{.99(1,1)} = 4052$ - - - - at 1% level

These critical values are larger than any observed F ratio of the __________ - order interaction, AxB, AxC, and BxC. Thus, F_{AxB}, F_{AxC}, and F_{BxC} are statistically

______________________.

$$\frac{2.042}{9.375} = 0.218$$

$$\frac{0.375}{9.375} = 0.040$$

$$\frac{1.042}{9.375} = 0.111$$

$$\frac{9.375}{0.667} = 14.055$$

---

(326) The critical F values which are to be compared with the observed F ratio

$F_{AxBxC} = 14.055$

are obtained from the Tables of F and shown below.

$F_{.95(1,16)} = 4.49$ - - - - - - at 5% level

$F_{.99(1,16)} = 8.53$ - - - - - at 1% level

$F_{.999(1,16)} = 16.12$ - - - - at 0.1% level

By comparing these critical values with the observed value, we conclude that F_{AxBxC} is __________ at _____% level.

first

not significant

(327) These results are entered in the Table of ANOVA.

Table 25-[illegible] F-ratio (All Random)

Source of Variation	Sum of Squares	Degree of Freedom	Variance estimate	F-ratio
A	5.042	1	5.042	
B	92.042	1	92.042	
C	0.375	1	0.375	
AxB	2.042	1	2.042	0.218
AxC	0.375	1	0.375	0.040
BxC	1.042	1	1.042	0.111
AxBxC	9.375	1	9.375	14.055**
Residual	10.667	16	0.667	
Total	120.958	23		

Notice that since we have no appropriate ______ for the three main effects we have no F-ratios for them.

significant

---

(328) Now suppose that we have not randomly selected the levels a_1 and a_2 for the A factor; b_1 and b_2 for the B factor; c_1 and c_2 for the C factor. We have this time a so-called ______ ANOVA model.

error term
(or error
variance estimate)

(329) To determine the appropriate error variance estimate for an F test for a particular source of variation in the fixed ANOVA model the following expectations (or components) of variance estimates are to be considered.

fixed

Table 28-a Expectations of Variance Estimates
A, B, & C All Fixed

Source	Expectations of Variance Estimate
A	$\sigma^2 + nbc\theta_a^2$
B	$\sigma^2 + nac\theta_b^2$
C	$\sigma^2 + nab\theta_c^2$
AxB	$\sigma^2 + nc\theta_{ab}^2$
AxC	$\sigma^2 + nb\theta_{ac}^2$
BxC	$\sigma^2 + na\theta_{bc}^2$
AxBxC	$\sigma^2 + n\theta_{abc}^2$
Residual	σ^2

In the table,

σ^2 = error variance

$nbc\theta_a^2$ = variance directly attributable to the ______________ effect

Notice that for the fixed ANOVA model, each source of variation has only two components. One is the ____________ and the other is the variance diredtly attributable to its own effect.

(330) To test the significance of a given effect with an F-test, an appropriate error variance estimate must be selected. To do this you should compare the components of the variance estimate of the given effect with that of a variance estimate in Table 28-a Frame 329. The variance estimate which has all the components of the given effect _________ that component which is directly attributable to the effect in question is the appropriate error variance estimate.	A error variance
(331) Since the components of the variance estimate of the A effect consist of $\sigma_{\sim}^{2}$ and $nbc\theta_a^2$, the variance estimate of the appropriate error term must have the component of ________________. Thus, from Table 28-a, the appropriate error term for the A effect is the ____________ variation.	except

(332) In effect the residual variation is the appropriate ______________ against which the rest of the variance estimates of the fixed ANOVA model may be tested. Thus, we have the following table.

Table 28-b Error Variance Estimates A, B, & C All Fixed

Source	Error term for F test
A	Residual
B	Residual
C	Residual
AxB	Residual
AxC	Residual
BxC	Residual
AxBxC	Residual

σ^2

residual

(333) Based on this we can now compute seven observed F ratios:

$$F_A = \frac{s^2_A}{s^2_{resid}}; \qquad F_B = \frac{s^2_B}{s^{S\,2}_{resid}};$$

$$F_C = \frac{s^2_C}{s^2_{resid}}; \qquad F_{AxB} = \frac{s^2_{AxB}}{s^2_{resid}};$$

$$F_{AxC} = \frac{s^2_{AxC}}{s^2_{resid}}; \qquad F_{BxC} = \frac{s^2_{BxC}}{s^2_{resid}};$$

$$F_{AxBxC} = \frac{s^2_{AxBxC}}{s^2_{resid}};$$

Substituting

$s^2_A = 5.042$

$s^2_B = 92.042$

$s^2_C = 0.375$

$s^2_{AxB} = 2.042$

$s^2_{AxC} = 0.375$

$s^2_{BxC} = 1.042$

$s^2_{AxBxC} = 9.375$

$s^2_{resid} = 0.667$

in the formulas, we have

$F_A =$ ______________

$F_B =$ ______________

$F_C =$ ______________

$F_{AxB} =$ ______________

$F_{AxC} =$ ______________

$F_{BxC} =$ ______________

$F_{AxBxC} =$ ______________

error term

(33u) The critical F values to be compared with these observed F values are obtained from the Tables of F and shown below:

$F_{.95(1,16)} = 4.49$ - - - - - - - - at 5% level

$F_{.99(1,16)} = 8.53$ - - - - - - - - at 1% level

$F_{.999(1,16)} = 16.12$ - - - - - - - at 0.1% level

Those observed F values which exceed one or all of these critical values are ____________; those which do not exceed these critical values are ____________________ .

$\frac{5.042}{0.667} = 7.559$

$\frac{92.042}{0.667} = 137.994$

$\frac{0.375}{0.667} = 0.562$

$\frac{2.042}{0.667} = 3.061$

$\frac{0.375}{0.667} = 0.562$

$\frac{1.042}{0.667} = 1.562$

$\frac{9.375}{0.667} = 14.055$

(33v) The results are shown in the table of ANOVA as follows:

Table 25-f F-Ratio (VII Fixed)

Source of variation	Sum of Squares	Degree of freedom	Variance estimate	F-ratio
A	5.042	1	5.042	7.559*
B	92.042	1	92.042	137.994***
C	0.375	1	0.375	0.562
AxB	2.042	1	2.042	3.061
AxC	0.375	1	0.375	0.562
BxC	1.042	1	1.042	1.562
AxBxC	9.375	1	9.375	14.055**
Residual	10.665	16	0.667	
Total	120.958	23		

Notice that the F ratios marked with astericks are ____________; with * indicates the 5% level, ** denote 1% level, *** represent 0.1% level.

significant

not significant

significant

(336) There are three ANOVA models. We have considered the random model and the fixed model so far. The third one is referred to as the mixed model in which the levels (or treatments) of some factors are randomly selected and the levels of others are not. In our present example, suppose that we have randomly selected levels c_1 and c_2 for the factor C but levels a_1 and a_2 for the factor A and b_1 and b_2 for the factor B have not been selected at random. We now have a ______________ ANOVA model.

mixed

(337) To determine the appropriate error terms of this model, the following expectations (or components) of variance estimates should be investigated.

Table 29-a Expectations of Variance Estimates
A and B Fixed, C Random

Source	Expectation of Variance Estimate
A	$\tilde{\sigma}^2 + nb\tilde{\sigma}^2_{ac} + nbc\theta^2_a$
B	$\tilde{\sigma}^2 + na\tilde{\sigma}^2_{bc} + nac\theta^2_b$
C	$\tilde{\sigma}^2 + nab\tilde{\sigma}^2_c$
AxB	$\tilde{\sigma}^2 + n\tilde{\sigma}^2_{abc} + nc\theta^2_{ab}$
AxC	$\tilde{\sigma}^2 + nb\tilde{\sigma}^2_{ac}$
BxC	$\tilde{\sigma}^2 + na\tilde{\sigma}^2_{bc}$
AxBxC	$\tilde{\sigma}^2 + n\tilde{\sigma}_{abc}{}^2$
Residual	$\tilde{\sigma}^2$

In this table, $nbc\theta^2_a$ = variance directly attributable to the ________ effect.

$\tilde{\sigma}^2$ = ________________

A

error variance

(332) It must be mentioned that this table is constructed by deleting certain components of the variance estimates of the random ANOVA model (Table 27-a, Frame 319). For instance, in Table 27-a the components of variance estimates for the A effect are given as follows:

$$\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab} + nbc\sigma^2_{a}$$

Take one component at a time, except error variance (σ^2) and the variance directly attributable for the A effect ($nbc\sigma^2_a$), say $n\sigma^2_{abc}$.

first delete subscript a which is necessary to describe the A effect. We now have $n\sigma^2_{bc}$.

If any one of the remaining subscripts represents a fixed effect, this component is deleted. Since b denotes the _________ effect B, component _____ is deleted from the expectation. Similarly component $nc\sigma^2_{ab}$ is deleted. Thus, for the mixed ANOVA model the components of variance estimates of the A effect become:

$$\sigma^2 + nb\sigma^2_{ac} + nbc\theta^2_{a}$$

Notice that $nbc\theta^2_a$ indicate the component of a fixed effect A while $nbc\sigma^2_a$ represent the component of a ______ effect A.

(339) The appropriate error term for a given effect is the one whose components of variance estimate include all of the components of variance estimate of the given effect ______ ________ the component directly attributable to this very effect. Therefore, the appropriate error term for the A effect (see Table 29-a, Frame 337) is the ________________ interaction.

A: $\sigma^2 + nb\sigma_{ac}^2 + nbc\theta_a^2$

______: $\sigma^2 + nb\sigma_{ac}^2$

fixed

$n\sigma_{abc}^2$

random

---

(340) The error terms for this mixed ANOVA model are as follows:

Table 29-b. Error Variance Estimates
A and B Fixed, C Random

Source	Error term for F test
A	AxC
B	BxC
C	Residual
AxB	AxBxC
AxC	Residual
B xC	Residual
AxBxC	Residual

To obtain an observed F for the A effect, the A variance estimate must be tested against the ________________ variance estimate

except

AxC

AxC

(341) That is,

$$F_A = \frac{s^2_A}{s^2_{AxC}} \text{ - - - - - - - - - - (III-51)}$$

Substituting $s^2_A = 5.042$ and $s^2_{AxC} = 0.375$ in the formula, we have:

F_A = ________________

AxC

---

(342) The remaining F-ratios can now be computed by the formulas

$$F_B = \frac{s^2_B}{s^2_{BxC}} \text{ - - - - - - - - - (III-52)}$$

$$F_C = \frac{s^2_C}{s^2_{resid}} \text{ - - - - - - - - (III-53)}$$

F_{AxB} = __________ - - - - - - - - (III-54)

F_{AxC} = __________ - - - - - - - - (III-55)

F_{BxC} = __________ - - - - - - - - (III-56)

F_{AxBxC} = __________ - - - - - - - - (III-57)

$\frac{5.042}{0.375} = 13.445$

(345) Substituting

$$s^2_B = 92.042$$

$$s^2_C = 0.375$$

$$s^2_{AxB} = 2.042$$

$$s^2_{AxC} = 0.375$$

$$s^2_{BxC} = 1.042$$

$$s^2_{AxBxC} = 9.375$$

$$s^2_{resid} = 0.667$$

in the formulas, we have

F_B = ____________________

F_C = ____________________

F_{AxB} = ____________________

F_{BxC} = ____________________

F_{AxBxC} = ____________________

$$\frac{s^2_{AxB}}{s^2_{AxBxC}}$$

$$\frac{s^2_{AxC}}{s^2_{resid}}$$

$$\frac{s^2_{BxC}}{s^2_{resid}}$$

$$\frac{s^2_{AxBxC}}{s^2_{resid}}$$

---

(346) The critical values: $F_{.95(1, 16)} = 4.49$, $F_{.99(1, 16)} = 8.53$ and $F_{.999(1, 16)} = 16.12$ should be compared with F_C, ________, ________ and ________ whereas $F_{.95(1, 1)} = 161$ and $F_{.99(1, 1)} = 4,052$ should be compared with F_A, ________, and ________. If the observed F-ratio exceeds the critical F value the effect is statistically ____________.

$$\frac{92.042}{1.042} = 88.332$$

$$\frac{0.375}{0.667} = 0.562$$

$$\frac{2.042}{9.375} = 0.218$$

$$\frac{0.375}{0.667} = 0.562$$

$$\frac{1.042}{0.667} = 1.562$$

$$\frac{9.375}{0.667} = 14.055$$

(345) The Table of ANOVA now looks like this:

Table 25-g F-Ratio (VII A&B Fixed; C Random)

Source of variation	Sum of Squares	Degree of Freedom	Variance Estimate	F-Ratio
A	5.042	1	5.042	13.445
B	92.042	1	92.042	88.332
C	0.375	1	0.375	0.562
AxB	2.042	1	2.042	0.218
AxC	0.375	1	0.375	0.562
BxC	1.042	1	1.042	1.562
AxBxC	9.375	1	9.375	14.055
Residual	10.665	16	0.667	
Total	120.958	23		

Finally, we conclude by saying that the

________ interaction is significant.

F_{AxC}, F_{BxC}, and F_{AxBxC}

F_B and F_{AxB}

significant

---

(346) Summary The total variation of a 2x2x2 factorial experiment (with A, B, and C factors) is partitioned into three ________ effects, three ________ -order interactions, one ________ -order interaction, and the ________ variation.

AxBxC

(347) Three different methods can be used for computation of the sums of squares for each source of variation. They are (1) the conventional formula (2) the short-cut formula, and (3) the so-called "__________ ". With the short-cut formula the sum of squares of a main effect, say A, is computed by:

$$SS_A = \frac{(A_1 - A_2)^2}{N}$$

where A_1 = the a_1 __________

N = __________ of scores

main effects
(A, B, and C)

first-order interactions
(AxB, AxC, & BxC)

second-order interactions
(AxBxC)

residual

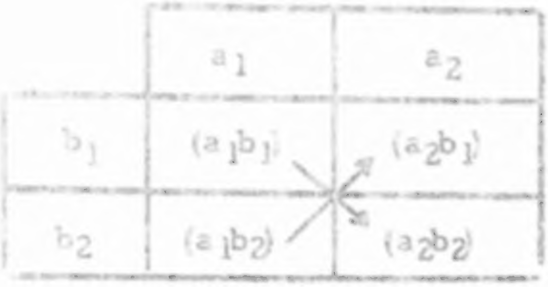
---

(348) With the short-cut formula the first order interaction, say AxB, is computed by

$$SS_{A\times B} = \frac{1}{N}\left[(a_1b_1 + a_2b_2) - (a_1b_2 + a_2b_1)\right]^2$$

where N = __________ of scores

a_1b_1 = the a_1b_1 __________



	a_1	a_2
b_1	(a_1b_1)	(a_2b_1)
b_2	(a_1b_2)	(a_2b_2)

orthogonal comparisons

level total

total number

(349) With the short cut formula the second-order interaction AxBxC is computed as follows:

$$SS_{AxBxC} = \frac{1}{N}(T_1 - T_2)^2$$

	a_1		a_2		
	c_1	c_2	c_1	c_2	
b_1	$(a_1b_1c_1)$	$(a_1b_1c_2) \rightarrow$	$(a_2b_1c_1)$	$(a_2b_1c_2) \rightarrow$	T_1
b_2	$(a_1b_2c_1)$	$(a_1b_2c_2) \rightarrow$	$(a_2b_2c_1)$	$(a_2b_2c_2) \rightarrow$	T_2

where N = total number of scores

$T_1 = a_1b_1c_1 +$ ______ + ______ + ______

$T_2 = a_1b_2c_1 +$ ______ + ______ + ______

total number

cell total

$a_1b_2c_2 + a_2b_2c_1 + a_2b_1c_2$

$a_1b_1c_2 + a_2b_1c_1 + a_2b_2c_2$

---

(350) All of the sums of squares can be obtained by "orthogonal comparisions".

Orthogonal Comparisons For 2x2x2 Factorial

Effect	Treatment Combination							
	$a_1b_1c_1$	$a_1b_2c_1$	$a_1b_1c_2$	$a_1b_2c_2$	$a_2b_1c_1$	$a_2b_2c_1$	$a_2b_1c_2$	$a_2b_2c_2$
A	-	-	-	-	+	+	+	+
B	-	+	-	+	-	+	-	+
C	-	-	+	+	-	-	+	+
AxP	+	-	+	-	-	+	-	+
AxC	+	+	-	-	-	-	+	+
BxC	+	-	-	+	+	-	-	+
AxBxC	-	+	+	-	+	-	-	+

To obtain the sum of square for a specific effect, the eight treatment combinations are algebraically added in accorcance with the signs shown for that effect, and the result is squared and further __________ by the total number of scores.

For instance, for the A effect:

$$SS_A = \frac{1}{N}(-a_1b_1c_1 - a_2b_2c_1 - a_1b_1c_2 - a_1b_2c_2 + a_2b_1c_1 + a_2b_2c_1 + a_2b_1c_2 + a_2b_2c_2)^2$$

(351) The total number of degrees of freedom is ______________ less than the total number of scores in the experiment. Let N be the total number of score. Then, the total number of degrees of freedom is obtained by the formula

$$df_{total} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_$$

divided

(352) Since our experiment is a 3 x 3 x 3 factorial, each of the three factors has ______________ levels. Therefore, the number of degrees of freedom for each of the three factors is ______________. That is,

$$df_A = df_B = df_C = \_\_\_\_\_\_\_\_\_\_\_\_$$

one

N-1

(353) Knowing $df_A = df_B = df_C = 1$, the degrees of freedom for the three first-order interactions can be obtained as follows: $df_{A \times B} = df_A \times df_B =$ ______________ $df_{A \times C} = df_A \times df_C =$ ______________ $df_{B \times C} = df_B \times df_C =$ ______________	two one 1
(354) Similarly the number of degrees of freedom for the second-order interaction can be computed as follows: $df_{A \times B \times C} = df_A \times df_B \times df_C =$ __________	$(1)(1) = 1$ $(1)(1) = 1$ $(1)(1) = 1$

(355) Knowing the following

$df_{A} = df_{B} = df_{C} = df_{A \times B} = df_{A \times C}$

$= df_{B \times C} = df_{A \times B \times C} = 1,$

$df_{total} = N-1,$ and

$df_{total} = df_{A} + df_{B} + df_{C} + df_{AxB} + df_{AxC}$

$+ df_{BxC} + df_{AxBxC} + df_{resid}$

The residual degrees of freedom can be expressed as follows.

$df_{resid} = df_{total} - df_{A} - df_{B} - df_{C} - df_{AxB}$

$- df_{AxC} - df_{BxC} - df_{AxBxC}$

= ______________________________

(1)(1)(1) = 1

---

(356) As we have seen again and again, the variance estimate, known as the ______________, is obtained by dividing the ______________ by its corresponding ______________. We thus obtain eight variance estimates:

s^2_{A}, s^2_{B}, s^2_{C}, $s^2_{A \times B}$, $s^2_{A \times C}$, $s^2_{B \times C}$, s^2_{AxBxC}, and s^2_{resid}.

(N-1)-1-1-1-1-1-1-1

= N-8

(357) To obtain an F-ratio for an effect we must determine an appropriate error variance estimate against which the variance estimate of that effect may be tested. For the random ANOVA model in which the levels of all the three factors of the 2 x 2 x 2 factorial are __________ selected, the expectations (or components) of variance estimates are as follows.

Source	Expectation of Variance Estimate
A	$\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac} + nc\sigma^2_{ab} + nbc\sigma^2_{a}$
B	$\sigma^2 + n\sigma^2_{abc} + na\sigma^2_{bc} + nc\sigma^2_{ab} + nac\sigma^2_{b}$
C	$\sigma^2 + n\sigma^2_{abc} + na\sigma^2_{bc} + nb\sigma^2_{ac} + nab\sigma^2_{c}$
AxB	$\sigma^2 + n\sigma^2_{abc} + nc\sigma^2_{ab}$
AxC	$\sigma^2 + n\sigma^2_{abc} + nb\sigma^2_{ac}$
BxC	$\sigma^2 + n\sigma^2_{abc} + na\sigma^2_{bc}$
AxBxC	$\sigma^2 + n\sigma^2_{abc}$
Residual	σ^2

According to this table there is no appropriate error term for effects ________, ______________, and ____________.

Source AxBxC is an appropriate error term for __________, ____________, and ____________, residual is an appropriate error term for ______________.

mean square

sum of squares

number of degrees of freedom

(358) For the fixed ANOVA model in which the levels of the three factors of the 2 x 2 x 2 factorial are _______________ selected, the expectations (or components) of variance estimates are as follows:

Source	Expectations of variance estimate
A	$\sigma^2 + nbc\,\theta^2_a$
B	$\sigma^2 + nac\,\theta^2_b$
C	$\sigma^2 + nab\,\theta^2_c$
AxB	$\sigma^2 + nc\,\theta^2_{ab}$
AxC	$\sigma^2 + nb\,\theta^2_{ac}$
BxC	$\sigma^2 + na\,\theta^2_{bc}$
AxBxC	$\sigma^2 + n\,\theta^2_{abc}$
Residual	σ^2

According to this table, the _______________ variance estimate is the appropriate error term for the rest of the variance estimates.

randomly

A, B, and C

AxB, AxC, and BxC

A x B x C

(359) For a mixed ANOVA model in which factors A and B are fixed and factor C is random, the expectations (or components) of variance estimates are given as follows:

Source	Expectation of variance estimate
A	$\sigma^2 + nb\,\sigma^2_{ac} + nbc\,\theta^2_{a}$
B	$\sigma^2 + na\,\sigma^2_{bc} + nac\,\theta^2_{b}$
C	$\sigma^2 + nab\,\sigma^2_{c}$
AxB	$\sigma^2 + n\,{}^2_{abc} + nc\,\theta^2_{ab}$
AxC	$\sigma^2 + nb\,\sigma^2_{ac}$
BxC	$\sigma^2 + na\,\sigma^2_{bc}$
AxBxC	$\sigma^2 + n\,\sigma^2_{abc}$
Resid.	σ^2

According to this table, the residual variance estimate is the appropriate error term for ____________, ____________, ____________, and ____________; the AxBxC variance estimate is the appropriate error term for ____________; AxC is the error term for ____________; BxC is the error term for ____________.

not randomly

residual

(360) The observed F-ratio is then obtained by dividing the variance estimate of a source of variation by its corresponding ____________ variance estimate.

C, AxC, BxC, and

AxBxC

AxB

A

B

(361) The critical F values are next obtained from the Tables of F. The number of degrees of freedom for the given variance estimate gives the ____________ and the number of degrees of freedom for the error variance estimate gives the ____________ of the Tables of F in which the critical values are to be found.

error

(362) If the observed F value exceeds the critical value, the given source of variation is said to be ________________.

column

row

---

significant

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March, 1965

MULTIPLE COMPARISONS AMONG TREATMENTS

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TABLE OF CONTENTS

Page

Introduction (Frames 1 - 14) - - - - - - - - - - - - - 1

1. Duncan's Test for Equal n's - with Treatment Means (Frames 15 - 30) - - - - - - - - - - - 8

2. Duncan's Test for Equal n's - with Treatment Totals (Frames 31-39) - - - - - - - - - - - - 17

3. Scheffe's Test for Equal n's - with Treatment Means (Frames 40 - 48) - - - - - - - - - - - 23

4. Scheffe's Test for Equal n's - with Treatment Totals (Frames 49 - 55) - - - - - - - - - - - 31

5. Duncan's Test for Unequal n's - with Treatment Means (Frames 56 - 85) - - - - - - - - - - 38

6. Dunnett's Test For Equal n's - with Treatment Means (Frames 86 - 100) - - - - - - - - - - 54

(1) Experimental designs may be classified in terms of the number of experimental variables (factors, classification, or criteria) involved in the experiments. An experiment in which two or more factors (or classifications) involved is termed as a multifactor experiment. Each factor has two or more treatments (methods, groups, or whatever they may be.) Observations are made and are classified in accordance with the experimental variables (that is, factors or classifications). An experiment in which a set of K treatments are classified in accordance with a single factor is called a ________________ experiment.

---

(2) The primary objective of the single factor experiment is to compare the relative effectiveness of a set of K treatments. A null hypothesis, assuming that all of the K treatments do not differ significantly (in other words, they are all the same), is postulated and an attempt to reject this hypothesis is made. To accomplish this, the t-test is employed for $K=2$. When $K \geq 2$, the analysis of variance is utilized. Thus, for an experiment in which only two treatments are involved, ________________________ ____________ may be used to test the significance of difference between the two treatments.

single factor
(or single classification)

---

(3) In the case where $K \geq 3$, the analysis of variance is used. The analysis of variance may or may not reject the ________________ that assumes that all the K treatments do not differ significantly. A significant result revealed by the analysis of variance, however, will not tell how they differ. One particular treatment mean may differ significantly from all other treatment means, or some of the treatment means may differ significantly from each other but not from still others, and so on and so forth. It is obvious that we need some procedures by which to make further comparisons.

either the t-test
or analysis of
variance

(4) To illustrate our problem in question, let us consider a completely randomized single factor experiment in which there are five different treatments involved. The observed data of the experiment are given in the following table. Since K=5>2:

Table 1-a Observed Data (I)

	Treatment				
	A	B	C	D	E
	33	68	35	50	70
	40	61	28	48	82
	56	72	24	52	72
	42	67	30	45	68
	38	65	32	43	85
	47	70	25	55	71
Sum	256	403	174	293	448
Mean	42.7	67.2	29.0	48.8	74.7

In this case, we should use the ______ ______ to test the significance of difference among the five treatment means.

null hypothesis

(5) Before carrying out an analysis of variance, the observed data are usually coded. We may subtract from each item of the data a guessed mean or we may divide each item by an appropriate number without disturbing the analysis of variance that follows. Let us code the observed data presented in Table 1-a by subtracting from each item a guessed mean of __________________ and rearrange the table such that the X's and X^2's appear in the same table, we have

analysis of variance

Table 1-b X, X^2, ΣX, & ΣX^2 (I)

A		B		C		D		E		Total	
n=6		n=6		n=6		n=6		n=6		N=30	
X	X^2	X	X^2	X	X^2	X	X^2	X	X^2	X	X^2
-17	289	18	324	-15	225	0	0	20	400		
-10	100	11	121	-22	484	-2	4	32	1024		
6	36	22	484	-26	676	2	4	22	484		
-8	64	17	289	-20	400	-5	25	18	324		
-12	144	15	225	-18	324	-7	49	35	1225		
-3	9	20	400	-25	625	5	25	21	441		
Sum -44	642	103	1843	-126	2734	-7	107	148	3898	74	9224

From the table, we obtain

the correction term = $\frac{(\Sigma X)^2}{N}$

= __________________

(6) In the completely randomized single factor experiment, the total variance can be partitioned into two independent components: (1) the between-treatment variation and (2) the ______________ variation. The latter is considered to be the experimental error in this case. Our Table of Analysis of Variance will look like this:

Table 2-a Source of Variation

Source of Variation	SS	df	S^2	F
Between treatments Within treatments				
Total				

4

50

$$\frac{(74)^2}{30} = 182.53$$

(7) In accordance with the procedures implied by Table 2-a, let us first compute the sums of squares (SS's).

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = \_\_\_\_\_\_\_\_\_\_$$

$$SS_{between} = \frac{1}{n}\left[(\Sigma X_a)^2 + (\Sigma X_b)^2 + (\Sigma X_c)^2 + (\Sigma X_d)^2 + (\Sigma X_e)^2\right] - \frac{(\Sigma X)^2}{N}$$

$$= \_\_\_\_\_\_\_\_\_\_$$

$$SS_{within} = SS_{total} - SS_{between} = \_\_\_\_\_\_\_\_\_\_$$

Table 2-b Sums of Squares

Source of Variation	Sum of Squares	df	s^2	F
Between T. Within T.	8213.14 828.33			
Total	9041.47			

within-treatment

(8) Next, let us determine the degrees of freedom (df's) for the corresponding sums of squares.

$df_{total} = N - 1 =$ ____________

$df_{between} = K - 1 =$ ____________

$df_{within} = df_{total} - df_{between} =$ ________

where N and K denote the total number of cases and the number of treatments, respectively.

Table 2-c Degrees of Freedom

Source of Variation	Sum of Squares	Degree of freedom	s^2	F
Between T. Within T.	8213.14 828.33	4 25		
Total	9041.47	29		

$9224 - 182.53 = 9041.47$

$\frac{1}{6}[(-44)^2+(103)^2+(-128)^2+(-7)^2+(148)^2] - 182.53 =$ 8213.14

$9041.47 - 8213.14 = 828.33$

---

(9) The variance estimate, or as is usually called the mean square, is obtained by dividing the sum of squares by its corresponding number of degrees of freedom. Thus,

$$s^2_{between} = \frac{SS_{between}}{df_{between}} = \underline{\qquad\qquad}$$

$$s^2_{within} = \frac{SS_{within}}{df_{within}} = \underline{\qquad\qquad}$$

Table 2-d Variance Estimates

Source of Variation	Sum of Squares	Degree of freedom	Variance Estimate	F
Between T. Within T.	8213.14 828.33	4 25	2053.29 33.13	
Total	9041.47	29		

$30 - 1 = 29$

$5 - 1 = 4$

$29 - 4 = 25$

(10) The observed F-ratio is obtained by dividing the between-treatment variance estimate by the error variance estimate. Since the within-treatment variance estimate, in this case, is considered to be the ______________________, we have

$$F_{ob} = \frac{S^2_{between}}{S^2_{within}} = \_\_\_\_\_\_\_\_\_\_\_\_$$

From the Table of F (consult your textbook), we find the critical values for df = 4 (check column 4 of the Table of F) and df = 25 (check row 25 of the table) as follows:

$F_{.95}(4,25) = 2.76$ for the 5% level

$F_{.99}(4,25) = 4.18$ for the 1% level

$F_{.999}(4,25) = 6.49$ for the 0.1% level

Comparing the observed F-ratio and the critical values, we have $F_{ob} = 61.98 > F_{.999}(4,25) = 6.49$. Thus, the null hypothesis that assumes that there is ______________ among the five treatment means are rejected at $P = 0.001$ (or 0.1%) level.

Table 2-e F-Ratio

Source of Variation	Sum of Squares	Degree of freedom	Variance Estimate	F-Ratio
Between T.	8213.14	4	2053.29	61.98***
Within T.	828.33	25	33.13	
Total	9041.47	29		

$$\frac{8213.14}{4} = 2053.29$$

$$\frac{828.33}{25} = 33.13$$

(11) In other words, the five treatment means, $X_a = 44.7$, $X_b = 67.2$, $X_c = 29.0$, $X_d = 48.8$ and $X_e = 74.7$ are significantly different. The results of the analysis, however, does not tell exactly how these treatment means differ. It does not tell as to which particular treatment mean differs significantly from which treatment mean or means, or which particular treatment mean differ significantly from the average of which means, a combination of two, three or four single treatments, and so on. These and other comparisons may be made in accordance with the purpose of the research and the interest of the researcher. We are, now, dealing with the problem of ______________ comparisons.

error variance

$$\frac{2053.29}{33.13} = 61.98$$

no difference

---

(12) For a moment, let us check how many comparisons are possible in our present example. Since we have five treatments A,B,C,D, and E, we may compare two treatments at a time, that is. A vs. B, A vs. C, A vs. D, A vs. E, B vs. C, B vs. D, B vs. E, C vs. D, C vs. E, and D vs. E -- a total of 10 pairs. We may compare a single treatment with the average effect of two other treatments combined. There are ______________ combinations of these, including A vs. B+C, A vs. B+D, A vs B+E, ------, and so on. We may compare a single treatment mean with the average effect of three other treatments combined. There are ______________ combinations of these, including A vs. B+C+D, A vs. B+C+E, A vs. B+D+E, A vs. C+D+E, ----- and so on. We may also want to compare one single treatment with the average effect of four other treatments combined. There are ______________ combinations of these, including A vs. B+C+D+E, B vs. A+C+E+D, -------, and so forth. We may go further and compare the average effect of two particular treatments combined with the average effect of two other treatments combined. There are ______________ combinations of these, including A+B vs. C+D, A+B vs. C+E, A+B vs. D+E,------ and so forth.

multiple

(13) Finally, we may compare the average effect of two particular treatments with the average effect of three other treatments combined. There are ______________ such combinations, including A+B vs. C+D+E, A+C vs. B+D+E, ------, and so forth. In our present example, then there are a total of ______________ possible comparisons.

30 ($5\,{}_4C_2$)

20 ($5\,{}_4C_3$)

5 ($5\,{}_4C_4$)

30 (${}_5C_2 \cdot {}_3C_2$)

---

(14) To carry out the multiple comparisons, one of the following procedures may be used:

Newman-Kewls Test,
Duncan's Multiple Range Tests,
Scheffé's Test,
Tukey's Tests,
and Dunnett's Test.

10 (${}_5C_2 \cdot {}_3C_3$)

105

---

(15) The Duncan's procedure is appropriate for testing pairs of treatments. Let us, first, use the Duncan's procedure to carry out the comparisons. As you recall we have five treatments with same sample size of n = __________. Now, the first step is to arrange the five treatment means (see Frame 4) in an increasing order of magnitude in a two-way table like this:

Table 3-a Duncan's Test (I-A)

Mean \ Mean	C 29.0	A 42.7	D 48.8	B 67.2	E 74.7
C 29.0					
A 42.7					
D 48.8					
B 67.2					
E 74.7					

1. Duncan's Test for Equal n's - with treatment means.

(16) The second step is to enter the differences between the treatment means in the table in such a way that the entry in row i, column j will represent $\overline{X}_j - \overline{X}_i$, where $\overline{X}_i$ and $\overline{X}_j$ denote the means appear on the i th row and j th column, respectively. Notice that only the upper triangular entries are shown in the table. You do not have to show the lower triangular entries because they are identical with those shown in the upper triangular matrix. Actually the main diagonal of the square matrix is the ________________ of symmetry.

6

Table 3-b Duncan's Test (I-A)

Diff	C	A	D	B	E
	29.0	42.7	48.8	67.2	74.7
C 29.0	-	13.7	19.8	38.2	45.7
A 42.7		-	6.1	24.5	32.0
D 48.8			-	18.4	25.9
B 67.2				-	7.5
E 74.7					-

(17) Step 3 has to do with the determination of the shortest significant ranges. The shortest significant range (R) is obtained by multiplying the significant studentized range for the number of steps between two particular means (r) by the standard error of the mean ($S_{\bar{x}}$) of the distribution in question. That is,

R = ________________.

axis

$r \cdot s_{\bar{x}}$

(18) The significant studentized ranges, r's, are obtained by consulting the Table of Significant Studentized Ranges for Duncan's New Multiple Range Test (see the appendix of your textbook). In order to accomplish this, we have to do three things: (1) choosing the level of significance, (2) choosing the number of degrees of freedom, and (3) choosing the numbers of steps between two treatment means. For our present purpose, let us choose $\alpha = 0.01$ (or 1%) as the level of significance. The number of degrees of freedom corresponding to the error variance is used in determining r's, the critical values of the significant studentized ranges. Since $df_{error} = 25$ (refer back to Frame 8) in our present example, we should check Row 25 of the Table. And, since we have five treatment means, we shall be dealing with pairs of means with 2, 3, 4, and 5 steps apart. Thus we should check Columns ______________ in the Table.

k df	2	3	4	5	6
2					
3					

(19) The appropriate portion of the Table of Significant Studentized Ranges for Duncan's New Multiple Range Test with $\alpha = 0.01$ is presented below.

Table 4 Significant Studentized Ranges

df \ K	2	3	4	5
24	3.956	4.126	4.239	4.322
30	3.889	4.056	4.168	4.250

Notice that in our present example df = 25. However, the Table does not give the critical values for df = 25. Thus, we shall have to obtain them by interpolation.

$$r_2 = 3.956 - \frac{25-24}{30-24}(3.956-3.889) = \_\_\_\_\_\_\_\_$$

for df = 25, K = 2

$$r_3 = 4.126 - \frac{25-24}{30-24}(4.126-4.056) = \_\_\_\_\_\_\_\_$$

for df = 25, K = 3

$$r_4 = 4.239 - \frac{25-24}{30-24}(4.239-4.168) = \_\_\_\_\_\_\_\_$$

for df = 25, K = 4

$$r_5 = 4.322 - \frac{25-24}{30-24}(4.322-4.250) = \_\_\_\_\_\_\_\_$$

for df = 25, K = 5

2, 3, 4, and 5

(20) The standard error of the mean is obtained by dividing the error standard deviation by square root of the number of cases in each group. Since the error variance s^2_{error} = 33.13 (See Frame 9), s_{error} = __________. Thus,

$$S_{\bar{x}} = \frac{s_{error}}{\sqrt{n}} = \_\_\_\_\_\_\_\_\_\_$$

3.94 [the same result can be obtained by calculating $3.889 + \frac{30-24}{30-25}(3.956-3.889)$]

4.11 [try this, too: $4.056 + \frac{30-24}{30-25}(4.126-4.056)$]

4.23 [also: $4.168 + \frac{30-24}{30-25} \times (4.239-4.168)$]

4.31 [or $4.250+\frac{30-24}{30-25}(4.322-4.250)$]

(21) Now, the shortest significant ranges, R's, can be obtained as follows;

$R_2 = r_2 \cdot S_{\bar{x}} =$ __________

$R_3 = r_3 \cdot S_{\bar{x}} =$ __________

$R_4 = r_4 \cdot S_{\bar{x}} =$ __________

$R_5 = r_5 \cdot S_{\bar{x}} =$ __________

To facilitate the comparisons that follow, these R's are entered in Table 3. The Table now looks like this:

Table 3-c Duncan's Test (I-A)

	C	A	D	B	E	Shortest significant range
Mean	29.0	42.7	48.8	67.2	74.7	
C 29.0	-	13.7	19.8	38.2	45.7	R_2=9.36
A 42.7		-	6.1	24.5	32.0	R_3=9.66
D 48.8			-	18.4	25.9	R_4=9.94
B 67.2				-	7.5	R_5=10.13
E 74.7					-	

$\sqrt{33.13}$ - 5.76

$\frac{\sqrt{33.13}}{\sqrt{6}} = \sqrt{5.5217} = 2.35$

(22) Step 4 has to do with the testing of differences between pairs of treatment means. Those differences which exceed their corresponding R's are significant; and those differences which do not exceed their corresponding R's are ____________________.
To do the testing (or comparisons), the rightmost column of the entries in Table 3 is taken, first. As shown in the following we would, then move from top to bottom.

Table 3-C

A	D	B	E	Shortest significant range
	48.8	67.2	74.7	
	19.8	38.2	45.7**	R_2=9.26
	6.1	24.5	32.0**	R_3=9.66
	..	18.4	25.9**	R_4=9.94
		-	7.5	R_5=10.13
			-	

3.94 x 2.35 = 9.26

4.11 x 2.35 = 9.66

4.23 x 2.35 = 9.99

4.31 x 2.35 = 10.13

---

(23) The first entry in the column shown above is 45.7. Since this is the difference between the means E and C which are 5 steps apart, we must use R_5 to test it. We find that $45.7 > R_5 = 10.13$. Thus, this difference is statistically ____________________. The difference 45.7 is, then, marked with two asterisks (**). The next entry, 32.0, is the difference between the means E and A which are ____________ steps apart. Thus, R_4 must be used for the comparison. Since $32.0 > R_4 = 9.94$, the difference is significant. Again, the difference, 32.0 is marked with ____________.
You are supposed to continue the testing until a difference which does not exceed its corresponding R (the shortest significant range) is reached. No further tests are made beyond that point. In this case, $25.9 > R_3 = 9.66$ (significant) and $7.5 < R_2 = 9.26$ (not significant). Since you have reached a difference (in this case 7.5) which does not exceed its corresponding R, the testing for the column is discontinued.

not significant

(24) You should now take the next column to the left. Moving from top to bottom, you would compare 38.2, 24.5, and 18.4 with ________, __________, and __________.

$38.2 > R_4 = 9.94$ (significant)
$24.5 > R_3 = 9.66$ (significant)
$18.4 > R_2 = 9.26$ (significant)

All the three entried in this column should be marked with ______________.

Table 3-C

C	A	D	B	E	Shortest significant range
	42.7	48.8	67.2	74.7	
	13.7	19.88	38.2**	45.7**	R_2=9.26
	-	6.1	24.5**	32.0**	R_3=9.66
		-	18.4**	25.9**	R_4=9.94
			-	7.5	R_5=10.13
				-	

significant

four

two asterisks (**)

[Conventionally, one asterisk (*) is used when the result is significant at the 5% level. Two asterisks(**) are used for the 1% level and three asterisks (***) for the 0.1% level of significance.]

---

(25) Now, you go to the next column. The entry 19.8 is the difference between the means D and C which are __________ steps apart, so R_3 must be used. Since 19.8 > R_3 = 9.66, the two means are significantly __________. The entry 6.1 is the difference between the means D and A which are two steps apart. Since 6.1 < R_2 = 9.26, the difference 6.1 is ________.

Table 3-C

C	A	D	B	E	Shortest significant range
	42.7	48.8	67.2	74.7	
	13.7	19.8**	38.2**	45.7**	R_2-9.26
	-	6.1	24.5**	32.0**	R_3-9.66
		-	18.4**	25.9**	R_4-9.94
			-	7.5	R_5-10.13
				-	

R_4, R_3, and R_2

two asterisks (**)

(26) In the next column, we find only one entry 13.7 which is the difference between the means ________ and ______. Since 13.7 > R_2 = 9.26, the two means are significantly ______________.

three

different

not significant

Table 3-C

C	A	D	B	E	Shortest significant range
29.0	42.7	48.8	67.2	74.7	
-	13.7**	19.8**	38.2**	45.7**	R_2 = 9.26
		6.1	24.5**	32.0**	R_3 = 9.66
		-	18.4**	25.9**	R_4 = 9.94
			-	7.5	R_5 = 10.13
				-	

We have just completed the testing of difference between pairs of treatment means (Step 4)

A and C

different

(27) Finally, the testing (comparison) is summarized schematically at the bottom of the table by underscoring those treatment means which do not differ significantly. The first insignificant difference you have found in the table is 7.5 which represents the difference between the means E and B. Thus, E and B at the bottom of the table are underlined by the same line. Next, another insignificant difference 6.1 is noticed. Since this is the difference between the means D and A, ________ and ________ at the bottom of the table is underlined.

Table 3-d Duncan's Test (I-A)

(k = 5, n = 6)

	C	A	D	B	E	Shortest significant range
Mean	29.0	42.7	48.8	67.2	74.7	
C 29.0	-	13.7**	19.8**	38.2**	45.7**	R_2 = 9.26
A 42.7		-	6.1	24.5**	32.0**	R_3 = 9.66
D 48.8			-	18.4**	25.9**	R_4 = 9.94
B 67.2				-	7.5	R_5 = 10.13
E 74.7					-	

C $\underline{\text{A} \quad \text{D}}$ $\underline{\text{B} \quad \text{E}}$

(28) To interpret the results of the comparisons, your attention is called to the underlines. Any pair of treatment means not underlined by the same line are significantly different; any pair of treatment means underlined by the same line are not significantly different. In our present example, we have $C \quad \underline{A \quad D} \quad \underline{B \quad E}$ Thus, C ______ different significantly from all other treatment means; A ______ different from D, but ______ different from B: - - - - - - - - -, and so on.	D A
(29) Duncan's multiple test is based upon the concept of protection levels. The k-mean protection level is given by $(1 - \alpha)^{k-1}$, where α denotes the level of significance and k represents the number of treatment means. In our present example, we have $k = 5$ and $\alpha = 0.01$ (see Frame 18). Thus, the protection level is $(1-0.01)^{5-1} \approx 0.96$. This means that the minimum probability of finding no erroneous significant differences between the pairs of the five treatment means is 0.96 (or 96%). When k (the number of means) increases, the magnitude of the protection level ______.	is is not is
(30) You have just covered the steps involved in Duncan's Test for equal n's with treatment means used as the initial data. You can still come up with the same results if the treatment totals are used instead of the treatment means. The following frames will lead you to that kind of testing. If you do not want to use the treatment totals for the multiple comparisons, you may skip these frames and go directly to Frame 39.	decreases

(31) Let us take the same experiment, with the data shown in Frame 4, as an example. The totals for the five treatments are as follows: $\sum A = 256$, $\sum B = 403$, $\sum C = 174$, $\sum D = 293$, and $\sum E = 448$. The sample size n is the ________ for the five treatments. The first step in the multiple comparisons is to arrange the five treatment totals in an ascending order of magnitude in a two-way table like this:

Table 5-a Duncan's Test (I-B)

Sum / Sum	C 174	A 256	D 293	B 403	E 448
C 174					
A 256					
D 293					
B 403					
E 448					

2. Duncan's Test for Equal n's -- with treatment totals

---

(32) Next, the differences between the treatment totals are entered in the table. Since the main diagonal of the square matrix is the axis of ________, only the upper half of the matrix should be shown. Thus, the table now looks like this:

Table 5-b Duncan's Test (I-B)

Diff.	C 174	A 256	D 293	B 403	E 448
C 174	-	82	119	229	274
A 256		-	37	147	192
D 293			-	110	155
B 403				-	45
E 448					-

same (n = 6)

(33) The determination of the shortest significant ranges follows next. The shortest significant range (R) is obtained by the following formula

$$R = r.\sqrt{n.s^2_{error}}$$

where R denotes the shortest significant range, r represents the significant studentized range to be obtained from tables prepared by Duncan, n represents the sample size, and s^2_{error} denotes the error variance estimate. So far, we know n = ________ and s^2_{error} = ________.

18

symmetry

---

(34) The significant studentized ranges, r's, can be obtained from the Table of Significant Studentized Ranges for Duncan's New Multiple Range Test (consult your textbook). Let us choose α = 0.01 (the 1% level of significance). Since the number of treatments in our example is k = 5, and the number of degrees of freedom for the experimental error is df_{error} = 25, you should check columns ________ and row ________ of the Table. As we have done in Frame 19, the Table does not give the critical values for df = 25, we have to compute r's for df = 25, based on the values given for df = 24 and df = 30. Thus,

Table 6 Significant Studentized Ranges

k / df	2	3	4	5
24	3.956	4.126	4.239	4.322
25	3.94	4.11	4.23	4.31
30	3.889	4.056	4.168	4.250

6

33.13 (See Frame 9 variance estimate s^2 is often called as mean square denoted by MS)

(36) From the above table, we obtain $r_2 = 3.94$, $r_3 = 4.11$, $r_4 = 4.23$ and $r_5 = 4.31$. We can now compute the shortest significant ranges, R's:

$R_2 = r_2 \sqrt{ns^2_{error}} =$ __________

$R_3 = r_3 \sqrt{ns^2_{error}} =$ __________

$R_4 = r_4 \sqrt{ns^2_{error}} =$ __________

$R_5 = r_5 \sqrt{ns^2_{error}} =$ __________

To facilitate the comparisons that will follow, these R's may be entered in Table 5 like this:

Table 5-c Duncan's Test (I-B)

Total	C 174	A 256	D 293	B 403	E 448	Shortest significant range
C 174	-	82	119	229	274	$R_2 = 53.6$
A 256		-	37	147	192	$R_3 = 57.9$
D 293			-	110	155	$R_4 = 59.6$
B 403				-	45	$R_5 = 60.8$
E 448					-	

2, 3, 4, and 5

25

(36) Next, you are supposed to compare the differences of totals with the shortest significant ranges, taking one column at a time. The entry at the upper right corner of the table, 274, is the difference between the total E and C which are __________ steps apart. Thus, ______ must be used to compare with 274. Since $274 > R_5 = 60.8$, the difference is significant. To indicate the significance at $\alpha = 0.01$ level, the symbol ________ is marked. Moving from top to bottom, similar comparisons are made until a difference which does not exceed its corresponding R is reached. For instance, the entry at the bottom of the same column, 45 is the difference between totals E and B which are ________ steps apart and thus compared with R_2. Since 45 does not exceed $R_2 = 55.6$, it is ________ ________. No further tests are made beyond this point in the column. The next column, to the left, is taken and similar procedures follow. After covering all the columns you will come up with the following results:

Table 5-d Duncan's Test (I-B)

	C	A	D	B	E	Shortest Significant Range
Total	174	256	293	403	448	
C 174	-	82**	119**	229**	274**	$R_2 = 55.6$
A 256		-	37	147**	192**	$R_3 = 57.9$
D 293			-	110**	155**	$R_4 = 59.6$
B 403				-	45	$R_5 = 60.8$
E 448					-	

$3.94\sqrt{6 \times 33.13} = 3.94 \times 14.099 = 55.6$

$4.11\sqrt{6 \times 33.13} = 4.11 \times 14.099 = 57.9$

$4.23\sqrt{6 \times 33.13} = 4.23 \times 14.099 = 59.6$

$4.31\sqrt{6 \times 33.13} = 4.31 \times 14.099 = 60.8$

(37) Finally, the comparisons are summarized schematically at the bottom of the table where those treatment totals which do not differ significantly are underlined. We have found two insignificant differences 45 and 37. Since they represent differences between totals E and B and totals D and A, ________ and ________ as well as ________ and ________ are underlined.

five

R_5

as

two

not significant

Table 5-e Duncan's Test (I-B)

Total	C 174	A 256	D 293	B 403	E 448	shortest significant range
C 174	-	82**	119**	229**	274**	R_2 = 55.6
A 256		-	37	147**	192**	R_3 = 57.9
D 293			-	110**	155**	R_4 = 59.6
B 403				-	45	R_5 = 60.8
E 448					-	

C A D B E

(38) To interpret the results, you are to follow this rule: any pair of treatment totals not underlined by the same line are ____________ different, any pair of treatment totals underlined by the same line are ____________ different. Thus, in this case, B differs ____________ from all other totals but E.

E (or B) and B (or E)

D (or A) and A (or D)

(39) As we have seen, the Duncan's test is designed for comparisons of pairs of treatment means (or totals). With the number of treatments k = 5 in our example, we have dealt with ${}_5C_2$ = 10 pairs of treatment means: A vs. B, A vs. C, A vs. D, A vs. E, B vs. C, B vs. D, B vs. E, C vs. D, C vs. E, and D vs. E. Actually, you may want to carry out comparisons beyond this. For instance, you may wish to compare A with the average effect of B and C, or you may wish to compare the combined effect of A and B with that of C and D, and so forth. The Duncan's test does not provide for comparisons like these. It is appropriate for making comparisons of ____________ of treatment means (or totals) only. To accomplish any and all comparisons between a set of k means, the Scheffe's test is available.

significantly

not significantly

significantly

(40) To use Scheffé's test we do not have to plan the comparisons in advance. Let us take the same example, as was given in Frame 3, and carry out all possible comparisons with the Scheffé's test. To do the comparisons we may use either the treatment means or treatment totals. Let us use the treatment means for comparisons. The first step is to enter all possible comparisons and treatment means in a two-way table like this:

Table 6-s Scheffé's Test (I-C)

	Mean	A	B	C	D	E
	Comparison	42.7	67.2	29.0	48.3	74.7
${}_5C_2$	A vs. B A vs. C ⋮					
$5\cdot{}_4C_2$	A vs. B+C ⋮					
$5\cdot{}_4C_3$	A vs. B+C+D ⋮					
$5\cdot{}_4C_4$	A vs. B+C+D+E ⋮					
${}_5C_2\cdot{}_3C_2$	A+B vs. C+D ⋮					
${}_5C_2\cdot{}_3C_3$	A+B vs. C+D+E ⋮					

There are a total of ${}_5C_2 + 5\cdot{}_4C_2 + 5\cdot{}_4C_3 + 5\cdot{}_4C_4 + {}_5C_2\cdot{}_3C_2 + {}_5C_2\cdot{}_3C_3 =$ ________ comparisons.

3. The Scheffé's Test for Equal n's — with treatment means.

pairs

(41) Step 2 is to enter coefficients of the treatment means in the table. Let d_i denote the comparison (difference), then the first comparison $d_1 = \bar{X}_a - \bar{X}_b$. Thus, the coefficient for $\bar{X}_a$ is equal to 1 and the coefficient for X_b is equal to -1. Similarly, the eleventh comparison $d_{11} = X_a - \frac{1}{2}(X_b + X_c)$. Thus, the coefficient for X_a is equal to ______ and the coefficient for X_b and X_c are ____________ and ____________, respectively. The table now looks like this:

Table 6-b Scheffe's Test (I-C)

Coeff.	A	B	C	D	E
	42.7	67.2	29.0	48.8	74.7
A vs. B	1	-1	0	0	0
A vs. C	1	0	-1	0	0
A vs. D	1	0	0	-1	0
A vs. E	1	0	0	0	-1
B vs. C	0	1	-1	0	0
B vs. D	0	1	0	-1	0
B vs. E	0	1	0	0	-1
C vs. D	0	0	1	-1	0
C vs. E	0	0	1	0	-1
D vs. E	0	0	0	1	-1
A vs. B+C	1	-1/2	-1/2	0	0
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

$$\frac{5!}{3!2!} + 5\,\frac{4!}{2!2!} + 5\,\frac{4!}{1!3!}$$

$$+ 5\,\frac{4!}{0!4!} + \frac{5!}{3!2!} \cdot \frac{3!}{1!2!}$$

$$+ \frac{5!}{3!2!} \cdot \frac{3!}{0!3!}$$

$$= \frac{5.4.3}{3.2.1} + 5\,\frac{4.3}{2.1} + 5 \cdot \frac{4}{1}$$

$$+ 5.1 + \frac{5.4}{2.1} \cdot \frac{3}{1}$$

$$+ \frac{5.4}{2.1} \cdot 1$$

$$= 10 + 30 + 20 + 5 + 30 + 10$$

$$= 105$$

(42) Next, the sum of the squares of coefficients for each comparison is entered in the column headed $\sum c_i^2$. Thus, the entry in the eleventh row can be obtained as follows:

$$(\quad)^2 + (\quad)^2 + (\quad)^2 = 1.5$$

The differences then are entered in the column headed d_i. Thus, the entry in the first row will be

$\bar{X}_a - \bar{X}_b = 42.7 - 67.2 = -24.5$ and the entry in the eleventh row of the column will be

____

Now, the table looks like this:

Table 6-c Scheffe's Test (I-C)

MEAN / Comparison	A	B	C	D	E	$\sum c_i^2$	d_i
	42.7	67.2	29.0	48.8	74.7		
A vs. B	1	-1	0	0	0	2	-24.5
A vs. C	1	0	-1	0	0	2	13.7
A vs. D	1	0	0	-1	0	2	- 6.1
A vs. E	1	0	0	0	-1	2	-32.0
B vs. C	0	1	-1	0	0	2	38.2
B vs. D	0	1	0	-1	0	2	18.4
B vs. E	0	1	0	0	-1	2	- 7.5
C vs. D	0	0	1	-1	0	2	-19.8
C vs. E	0	0	1	0	-1	2	-45.7
D vs. E	0	0	0	1	-1	2	-25.9
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 5.4
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 5.63
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-13.23
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	16.05
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	4.12
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

1

$-\frac{1}{2}$

$-\frac{1}{2}$

(43) Next, the standard error of comparison s_{di} is computed and entered in the column headed s_{di} in the table. s_{di} is given as follows:

$$s_{di} = \sqrt{\frac{s^2_{error}}{n} \cdot \Sigma a_i^2} \quad \text{where}$$

s^2_{error} denotes the error variance estimate, n represents the sample size, and Σa_i^2 is the sum of squares of coefficients. We already know $s^2_{error} = 33.13$ (see Frame 9) and n = 6. Thus, for the first comparison, A vs. B,

s_{di} = ____________

The results are shown in the following table.

Table 6-d Scheffe's Test (I-C)

Mean / Comparison	A 42.7	B 67.2	C 29.0	D 48.8	E 74.7	Σa_i^2	d_i	s_{di}
A vs. B	1	-1	0	0	0	2	-24.5	3.32
A vs. C	1	0	-1	0	0	2	13.7	3.32
A vs. D	1	0	0	-1	0	2	- 6.1	3.32
A vs. E	1	0	0	0	-1	2	-32.0	3.32
B vs. C	0	1	-1	0	0	2	38.2	3.32
B vs. D	0	1	0	-1	0	2	18.4	3.32
B vs. E	0	1	0	0	-1	2	- 7.5	3.32
C vs. D	0	0	1	-1	0	2	-19.8	3.32
C vs. E	0	0	1	0	-1	2	-45.7	3.32
D vs. E	0	0	0	1	-1	2	-26.0	3.32
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 5.4	2.88
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 5.63	2.71
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-12.23	2.63
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	16.05	2.35
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	4.12	2.14
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

$(1)^2+(-\frac{1}{2})^2+(-\frac{1}{2})^2$

$\bar{X}_a - \frac{1}{2}(\bar{X}_b+\bar{X}_c)$
$= 42.7-(67.2+29.0)$
$= -5.4$

(44) The values of t's are then, computed by $t_i = \frac{d_i}{s_{di}}$ and entered in the column headed t_i. Thus the first entry will be

$t_i =$ ______________________

and so on.

$$\sqrt{\frac{33.1[illegible]}{6} \cdot 2} = 3.32$$

Table 6-c Scheffe's Test (I-C)

Mean / Comparison	A 42.7	B 67.2	C 29.0	D 48.8	E 74.7	Σa_i^2	d_i	s_{di}	t_i
A vs. B	1	-1	0	0	0	2	-24.5	3.32	- 7.38
A vs. C	1	0	-1	0		2	13.7	3.32	4.13
A vs. D	1	0	0	-1		2	- 6.1	3.32	- 1.84
A vs. E	1	0	0	0		2	-32.0	3.32	- 9.64
B vs. C	0	1	-1	0	0	2	38.2	3.32	11.51
B vs. D	0	1	0	-1	0	2	18.4	3.32	5.54
B vs. E	0	1	0	0	-1	2	- 7.5	3.32	- 2.26
C vs. D	0	0	1	-1	0	2	-19.8	3.32	- 5.96
C vs. E	0	0	1	0	-1	2	-45.7	3.32	-13.77
D vs. E	0	0	0	1	-1	2	-25.9	3.32	- 7.8
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 5.4	2.88	- 1.88
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 5.63	2.71	- 2.08
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-12.23	2.63	- 4.65
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	16.05	2.35	6.83
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	4.12	2.14	1.93
.	.	.	.	.	.	.	.		
.	.	.	.	.	.	.	.		

Each of these t's has ____________ df, the number of degrees of freedom corresponding to s^2_{error} of the analysis of variance. In our example, $s^2_{within} = s^2_{error}$. Thus,

$df_{within} =$ ______________________.

(46) Next, you are supposed to obtain a critical t value from the Table of t. Let us take $\alpha = 0.01$ (the 1% level of significance). And since $df_{error} = df_{within} = 25$ in our example, you will have to check the entry of Row ______ and column headed ______ in the Table of t. Thus, we have the critical value t = ______.

Table 7 Table of t

α / df	0.45	0.4 - - - - -	0.05 - - -	0.01
1			6.314	31.821
2			2.920	6.965
25			1.708	2.485

$$\frac{-24.5}{3.32} = -7.38$$

25

$df_{error} = 25$

(see frame 8)

(48) Finally, the t's in Table 6-e are compared with the critical value t = 2.485. When the absolute value, that is the value without the minus sign, of any t in Table 6-e exceeds the critical value of 2.485, we conclude that this t value is ________ at the ______% level. We, then, mark the asterisks (**) to indicate the level of significance as follows:

25

0.01

2.485

Table 6-f Scheffe's Test (I-C)

Mean / Comparison	A 42.7	B 67.2	C 29.0	D 48.8	E 74.7	$\sum a_i^2$	d_i	s_{di}	t_i
A vs. B	1	-1	0	0	0	2	-24.5	3.32	- 7.38**
A vs. C	1	0	-1	0	0	2	13.7	3.32	4.13**
A vs. D	1	0	0	-1	0	2	- 6.1	3.32	- 1.84
A vs. E	1	0	0	0	-1	2	-32.0	3.32	- 9.64**
B vs. C	0	1	-1	0	0	2	38.2	3.32	11.51**
B vs. D	0	1	0	-1	0	2	18.4	3.32	5.54**
B vs. E	0	1	0	0	-1	2	- 7.5	3.32	- 2.26
C vs. D	0	0	1	-1	0	2	-19.8	3.32	- 5.96**
C vs. E	0	0	1	0	-1	2	-45.7	3.32	-13.77**
D vs. E	0	0	0	1	-1	2	-25.9	3.32	- 7.8 **
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 5.4	2.88	- 1.88
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 5.63	2.71	- 2.08
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-12.23	2.63	- 4.65**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	16.05	2.35	6.83**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	4.12	2.14	1.93
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Critical t value							t =		2.485

8

(47) As far as the comparisons between the ten pairs of treatment means are concerned, the results are the same with that carried out by the Duncan's Test (refer back to Frame 27). That is, the difference between A and D and the difference between B and E are

________________.

significant

1

(48) The differences between other possible comparisons have also been tested. For instance, the difference between A and the average effect of B and C proved to be ____________________; the difference between the average effect of A and B and the average effect of C and D proved to be ____________________.

not significant

(49) As we have seen, instead of using the treatment means, we may make comparisons in terms of the treatment totals. Let us use the treatment totals this time to carry out the multiple comparisons with the Scheffe's test. First, you are suppose to take the five treatment totals from Frame 3 and enter these values and all possible comparisons in a two-way table like this:

Table 8-a Scheffe's Test (I-D)

Total \ Total	A	B	C	D	E
Total	286	403	174	293	448
A vs. B					
A vs. C					
.					
.					
.					
A vs. B+C					
.					
.					
A vs. B+C+D					
.					
A vs. B+C+D+E					
.					
A+B vs. C+D					
.					
.					
A+B vs. C+D+E					
.					
.					
.					

Again, we have a total of $_5C_2 + 5 \cdot {_4C_2} + 5 \cdot {_4C_3} + 5 \cdot {_4C_4} + {_5C_2} \cdot {_3C_2} + {_5C_2} \cdot {_3C_3}$ = ____________________ comparisons.

4. The Scheffe's Test for Equal n's - with treatment totals.

not significant

significant

(50) Next, you are supposed to enter the coefficients of the treatment totals in the table. Let D_i represent the comparisons (difference) of treatment totals. Then, the first comparison shown in the table can be indicated by $D_1 = \Sigma X_a - \Sigma X_b$. Thus, the coefficient for ΣX_a is equal to 1 and the coefficient for ΣX_b is equal to -1. Similarly, the eleventh comparison (the eleventh row of the table) can be denoted by

$$D_{11} = \Sigma X_a - \tfrac{1}{2}(\Sigma X_b + \Sigma X_c).$$ Thus,

the coefficient for ΣX_a is __________,
the coefficient for ΣX_b is __________,
and the coefficient for ΣX_c is __________.

Table 8-b Scheffe's Test (I-D)

Coeff.	A	B	C	D	E
	256	403	174	293	445
A vs. B	1	-1	0	0	0
A vs. C	1	0	-1	0	0
A vs. D	1	0	0	-1	0
A vs. E	1	0	0	0	-1
B vs. C	0	1	-1	0	0
B vs. D	0	1	0	-1	0
B vs. E	0	1	0	0	-1
C vs. D	0	0	1	-1	0
C vs. E	0	0	1	0	-1
D vs. E	0	0	0	1	-1
A vs. B+C	1	-1/2	-1/2	0	0
.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0
.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4
.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0
.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3
.	.	.	.	.	.

$$\frac{5.4}{2.1} + 5\,\frac{4.3}{2.1} + 5\,\frac{4.3.2}{3.2.1}$$

$$+ 5\cdot 1 + \frac{5.4}{2.1}\cdot\frac{3.2}{2.1}$$

$$+ \frac{5.4}{2.1}\cdot(1)$$

$$= 10 + 30 + 20 + 5 + 30 + 1$$

$$= 105$$

(51) You are ready, now, to enter the sums of squares of coefficients and the differences of the treatment totals for all comparisons in the columns headed $\sum c_i^2$ and D_i in the table. The entries in the eleventh row can be obtained as follows:

$\sum c_{11}^2$ = ______________

D_{11} = ______________

Table 8-c Scheffe's Test (I-D)

Total / Comp.	A 256	B 403	C 174	D 293	E 448	$\sum c_i^2$	D_i
A vs. B	1	-1	0	0	0	2	-147
A vs. C	1	0	-1	0	0	2	82
A vs. D	1	0	0	-1	0	2	-37
A vs. E	1	0	0	0	-1	2	-192
B vs. C	0	1	-1	0	0	2	229
B vs. D	0	1	0	-1	0	2	110
B vs. E	0	1	0	0	-1	2	- 45
C vs. D	0	0	1	-1	0	2	-119
C vs. E	0	0	1	0	-1	2	-274
D vs. E	0	0	0	1	-1	2	-155
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 32.5
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	-34
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	- 73.5
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	96
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	24.5
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

(52) Next, the components, each with 1 df, of the treatment sum of squares, A's, are computed and entered in the column headed A_i. $A_i = \frac{D_i^2}{n \sum a_i^2}$ where D_i^2 denotes the square of difference between treatment totals, n represents the sample size, and $\sum a_i^2$ is the sum of squares of coefficients.

In our present example, for instance,

A_2 = ______________

$(1)^2 + (-\frac{1}{2})^2 + (-\frac{1}{2})^2$
$= 1.5$

$\sum X_a - \frac{1}{2}(\sum X_b + \sum X_c)$
$= 256 - \frac{1}{2}(403+174)$
$= -32.5$

Table 8-d Scheffe's Test (I-D)

Total	A	B	C	D	E	$\sum a_i^2$	D_i	A_i
Comp.	256	403	174	293	448			
A vs. B	1	-1	0	0	0	2	-147	1800.75
A vs. C	1	0	-1	0	0	2	82	560.33
A vs. D	1	0	0	-1	0	2	- 37	114.08
A vs. E	1	0	0	0	-1	2	-192	3072.00
B vs. C	0	1	-1	0	0	2	229	4370.08
B vs. D	0	1	0	-1	0	2	110	1008.33
B vs. E	0	1	0	0	-1	2	- 45	168.75
C vs. D	0	0	1	-1	0	2	-119	1180.08
C vs. E	0	0	1	0	-1	2	-274	6256.33
D vs. E	0	0	0	1	-1	2	-155	2002.08
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 32.5	117.36
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 34	144.50
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	- 73.5	720.30
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	96	1536.00
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	24.5	120.05
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

(63) The values of F's are, then computed by $F = \frac{A}{s^2_{error}}$, and entered in the column headed F_i. Thus,

$F_i =$ ______________________

and so on.

$\frac{82^2}{6 \times 2} = 560.33$

Table 3-e Scheffe's Test (I-D)

Total / Comp.	A 256	B 403	C 174	D 293	E 448	$\sum c_i^2$	D_i	A_i	F_i
A vs. B	1	-1	0	0	0	2	-147	1800.75	54.35
A vs. C	1	0	-1	0	0	2	82	560.33	16.91
A vs. D	1	0	0	-1	0	2	- 37	114.08	3.44
A vs. E	1	0	0	0	-1	2	-192	3072.00	92.73
B vs. C	0	1	-1	0	0	2	229	4370.08	131.91
B vs. D	0	1	0	-1	0	2	110	1008.33	30.44
B vs. E	0	1	0	0	-1	2	- 45	168.75	5.09
C vs. D	0	0	1	-1	0	2	-119	1180.08	35.62
C vs. E	0	0	1	0	-1	2	-274	6256.33	188.84
D vs. E	0	0	0	1	-1	2	-155	2002.08	60.43
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 32.5	117.36	3.54
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 34	144.50	4.36
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-73.5	720.30	21.74
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C-D	1/2	1/2	-1/2	-1/2	0	1	9.6	1536.00	46.36
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	24.5	120.05	3.62
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(54) Next, you are supposed to obtain a critical F value from the Table of F. This is an F-ratio for 1 df corresponding to the numerator and df_{error} corresponding to the denominator. Since $df_{error} = 25$ (see Frame 8) in our example, you will have to check the entry of Column ____ and Row ______ in the Table of F. Let us take $\alpha = 0.01$. Then we have the critical value

$$F_{.99(1,25)} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Table 9 Table of F for $\alpha = 0.01$

$n_2 \backslash n_1$	1	2	. . . .	24	. .	30
1				6234.		6258.
2				99.46		99.47
.						
.						
.						
.						
.						
.						
.						
25	7.77	5.57				
26	7.72	5.53				

$$\frac{1800.75}{33.13} = 54.35$$

(55) Finally, the F's in Table 8-e are compared with the critical value $F_{.99}(1,25) = 7.77$. Any F value which exceeds 7.77 is ____________ at the ________% level. Two asterisks are marked for each significant F. The result of comparisons is as follows:

1

25

7.77

Table 8-f Scheffe's Test (I-D)

Total	A	B	C	D	E	Σa_i^2	D_i	A_i	F_i
Comp.	256	403	174	393	448				
A vs. B	1	-1	0	0	0	2	-147	1800.75	54.35**
A vs. C	1	0	-1	0	0	2	82	560.33	16.91**
A vs. D	1	0	0	-1	0	2	- 37	114.08	3.44
A vs. E	1	0	0	0	-1	2	-192	3072.00	92.73**
B vs. C	0	1	-1	0	0	2	229	4370.08	13.91**
B vs. D	0	1	0	-1	0	2	110	1008.33	30.44**
B vs. E	0	1	0	0	-1	2	- 45	168.75	5.09
C vs. D	0	0	1	-1	0	2	-119	1180.08	35.62**
C vs. E.	0	0	1	0	-1	2	-274	6256.33	188.84**
D vs. E	0	0	0	1	-1	2	-155	2002.08	60.43**
A vs. B+C	1	-1/2	-1/2	0	0	1.5	- 32.5	117.36	3.54
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
A vs. B+C+D	1	-1/3	-1/3	-1/3	0	1.33	- 34	144.50	4.36
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
A vs. B+C+D+E	1	-1/4	-1/4	-1/4	-1/4	1.25	-73.5	720.30	21.74**
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
A+B vs. C+D	1/2	1/2	-1/2	-1/2	0	1	96	1536.00	46.36**
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
A+B vs. C+D+E	1/2	1/2	-1/3	-1/3	-1/3	0.83	24.5	120.06	3.62
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
Critical F Value							$F_{.99(1,25)} = 7.77$		

5. The Duncan's Test for unequal n's with treatment means.

(56) So far, we have dealt with multiple comparisons among treatments for the experiments with equal n's. Let us now consider those experiments with unequal n's (that is, unequal sample size). Suppose we have obtained the following data from a completely randomized single factor experiment with unequal n's. We may use the Duncan's Test, developed for the purpose, to carry out comparisons between ______________ of treatments.

Table 10-a Observed Data (II)

	Treatment					
	A	B	C	D	E	F
	56	48	72	72	70	35
	47	60	58	80	68	28
	52	55	65	85	75	32
	38	52	62	90	55	40
		40	75		60	24
			72		78	
Sum	193	255	404	327	406	159
Mean	48.25	51.00	67.34	81.75	67.67	31.80

significant

1

(57) As we have seen, prior to making comparisons, an analysis of variance is to be made. As usual, to simplify the computation that follows, let us code the observed data by subtracting from each score in Table 10-a a guessed mean of ______________ and rearrange the table in such a way that the coded X's and X^2's appear in the same table:

Table 10-b X, X^2, ΣX, & ΣX^2 (II)

	A $n_a=4$		B $n_b=5$		C $n_c=6$		D $n_d=4$		E $n_e=6$		F $n_f=5$		Total $N=30$	
	X	X^2	X	X^2	X	X^2	X	X^2	X	X^2	X	X^3	X	X^2
	1	1	-7	49	17	289	17	289	15	225	-20	400		
	-8	64	5	25	3	9	25	625	13	169	-27	729		
	-3	9	0	0	10	100	30	900	20	400	-23	529		
	-17	289	-3	9	7	49	35	1225	0	0	-15	225		
			-15	225	20	400			5	25	-31	961		
					17	289			23	529				
Sum	-27	363	-20	308	74	1136	107	3039	76	1348	-116	2844	94	9038

From the table, we obtain the correction term = $\frac{(\Sigma X)^2}{N}$ = ______________

pairs

(58) Since this is a completely randomized single factor experiment, the total variance can be partitioned into two independent components: (1) the between-treatment variation and (2) the within-treatment variation which is considered to be the ________ variation. So, the Table of Analysis of Variance looks like this:

Table 11-a Source of Variation

Source of Variation	SS	df	s^2	F
Between treatments Within treatments				
Total				

55

$$\frac{(94)^2}{30} = 294.53$$

---

(59) Let us, now, compute the sums of squares (SS's).

$$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = \_\_\_\_\_\_\_\_$$

$$SS_{between} = \left[\frac{(\Sigma X_a)^2}{n_a} + \frac{(\Sigma X_b)^2}{n_b} + \frac{(\Sigma X_c)^2}{n_c} + \frac{(\Sigma X_d)^2}{n_d} + \frac{(\Sigma X_e)^2}{n_e} + \frac{(\Sigma X_f)^2}{n_f} \right] - \frac{(\Sigma X)^2}{N} = \_\_\_\_\_\_\_\_$$

$$SS_{within} = SS_{total} - SS_{between} = \_\_\_\_\_\_\_\_$$

Table 11-b Sums of Squares

Source of Variation	Sum of squares	df	s^2	F
Between T. Within T.	7396.51 1346.96			
Total	8743.47			

experimental error
(or error)

(60) The degrees of freedom (df's) associated with the respective sums of squares are to be determined, next.

$df_{total} = N-1 =$ __________

$df_{between} = k-1 =$ __________

$df_{within} = df_{total} - df_{between} =$

____

Where N represents the total numbers of cases and k denotes the number of treatments in the experiment.

Table 11-C Degrees of Freedom

Source of Variation	Sum of Squares	Degree of freedom	s^2	F
Between T.	7396.51	5		
Within T.	1346.96	24		
Total	8743.47	29		

(61) Two variance estimates, or mean squares, based on the between-treatment and within-treatment variations are to be computed. The variance estimate is obtained by dividing the sum of squares by its corresponding number of degrees of freedom. Thus,

$s^2_{between} =$ __________

$s^2_{within} =$ __________

Table 11-d Variance Estimates

Source of variation	Sum of Squares	Degree of freedom	Variance Estimate	F
Between T.	7396.51	5	1479.30	
Within T.	1346.96	24	56.12	
Total	8743.47	29		

$9038 - 294.53 = 8743.47$

$$\left[\frac{(-27)^2}{4} + \frac{(-20)^2}{5} + \frac{(74)^2}{6} + \frac{(107)^2}{4} + \frac{(76)^2}{6} + \frac{(-116)^2}{5}\right]$$

$-294.53 = 7691.04 - 294.53$
$= 7396.51$

$8743.47 - 7396.51 = 1346.96$

$30 - 1 = 29$

$6 - 1 = 5$

$29 - 5 = 24$

(62) Next, the observed F-ratio is obtained by dividing the between-treatment variance estimate by the error variance estimate. Thus,

$$F_{ob} = \frac{s^2_{between}}{s^2_{error}} = \_\_\_\_\_\_\_\_\_\_\_\_$$

From the Table of F (consult your textbook) we find the critical values for df = 5 (check column 5 of the Table of F) and df = 24 (check Row 24 of the same table) as follows:

$F_{.95(5,24)} = 2.62$ for the 5% level

$F_{.99(5,24)} = 3.90$ for the 1% level

$F_{.999(5,24)} = 5.98$ for the 0.1% level

Let us compare the observed F-ratio and the critical values. We find

$$F_{ob} = 26.36 > F_{.999(5,24)} = 5.98$$

Therefore, the null hypothesis that assumes that there is no difference among the five treatments are __________ at the 0.1% level.

Table 11-a F-Ratio

Source of Variation	Sum of Squares	Degree of Freedom	Variance Estimate	F-ratio
Between T.	7396.51	5	1479.30	26.36***
Within T.	1346.96	24	56.12	
Total	8743.47	29		

$$\frac{7396.51}{5} = 1479.30$$

$$\frac{1346.96}{24} = 56.12$$

(63) Now you are supposed to rank the five treatment means in ascending order in a two-way table like this (numbers in parentheses show replications):

Table 12-a Duncan's Test (II)

	F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75
Mean						
F31.80						
A48.25						
B51.00						
C67.34						
E67.67						
D81.75						

And then differences between pairs of means are entered in the table such that the entry in ith column and jth row will be equal to the difference between the mean in ____th column and the mean in ____ th row. Thus, we have

Table 12-b Duncan's Test (II-A)

Diff.	F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75
F 31.80	-	16.45	19.20	35.54	35.87	49.95
A 48.25		-	2.75	19.09	19.42	33.50
B 51.00			-	16.34	16.67	30.75
C 67.34				-	0.33	14.41
E 67.67					-	14.08
D 81.75						-

$$\frac{1479.30}{56.12} = 26.36$$

rejected

(64) Next, you are to compute adjusted differences based on the results shown in Table 12-b and the following formula:

$$D' = D\sqrt{\frac{3r_i \cdot r_j}{r_i + r_d}}$$

where D' = adjusted difference,
D = difference between two means,
r_i = replication number (or number of cases) for the ith mean, and
r_j = replication number (or number of cases) for the jth mean.

Thus,

$$D'_{df} = D_{df}\sqrt{\frac{2\,(4)(5)}{4+5}} = 49.95\sqrt{\frac{40}{9}} = \underline{\hspace{3em}}$$

D'_{da} = ______________________,

D'_{db} = ______________________,

D'_{de} = ______________________,

etc.

These adjusted differences are then entered in the table as follows:

Table 12-c Duncan's Test (II-A)

Adj. Diff.	F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75
F 31.80	–	34.68	42.93	82.99	83.76	105.29
A 48.25		–	5.80	41.81	42.53	67.00
B 51.00			–	38.15	38.92	64.82
C 67.34				–	0.81	31.56
E 67.67					–	30.83
D 81.75						–

i (or j)

j (or i)

(65) Critical ranges R's are to be determined next. First, you should obtain the studentized significant ranges (r's) from the Table of significant studentized Ranges for Duncan's New Multiple Range Test provided by your textbook. Let us take the level of significance $\alpha = 0.001$. According to the results of our analysis of variance (see Frame 60) $df_{error} = df_{within} = 24$. Thus, we have

$r_2 =$ ____________________

$r_3 =$ ____________________

$r_4 =$ ____________________

$r_5 =$ ____________________

Table 13 Significant Studentized Ranges for $\alpha = 0.001$

df \ k*	2	3	4	5	6
24	5.297	5.484	5.612	5.708	5.784

*k denotes steps between two treatment means

105.29

$$33.50\sqrt{\frac{2(4)(4)}{4+4}} = 67.00$$

$$30.75\sqrt{\frac{2(4)(5)}{4+5}} = 64.82$$

$$14.41\sqrt{\frac{2(4)(6)}{4+6}} = 31.55$$

(66) Critical ranges R's are then obtained by multiplying the r's by the square root of error variance estimate (or mean square) found by the analysis of variance (see Frame 61) That is,

$$R' = r\sqrt{s^2_{error}} = r \cdot s_{error}$$

Therefore,

$R'_2 = r_2 \cdot s_{error} =$ ________________

$R'_3 = r_3 \cdot s_{error} =$ ________________

$R'_4 = r_4 \cdot s_{error} =$ ________________

$R'_5 = r_5 \cdot s_{error} =$ ________________

These critical values are entered in the table as follows:

Table 12-d Duncan's Test (II)

Mean	F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical ranges
F 31.80	-	34.68	42.93	82.99	83.76	105.29	$R'_2 = 39.67$
A 48.25		-	5.80	41.81	42.53	67.00	$R'_3 = 41.08$
B 51.00			-	38.15	38.92	64.82	$R'_4 = 42.03$
C 67.34				-	0.81	31.55	$R'_5 = 42.75$
E 67.67					-	30.83	$R'_6 = 43.33$
D 81.75						-	

5.297

5.484

5.612

5.708

5.784

(67) Now, we are ready to test differences between pairs of treatment means. Those adjusted differences which exceed their corresponding R's are ____________; those adjusted differences which ________ exceed their corresponding R's are not significant. To do the testing, the rightmost column headed E in Table 12-d is taken first. You would then move from top to bottom taking one entry at a time.

Table 12-e Duncan's Test (II)

C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical ranges
83.09	83.76	105.29***	$R_2 = 39.67$
41.81	42.53	67.00***	$R_3 = 41.06$
36.15	38.92	64.82***	$R_4 = 42.03$
-	0.81	31.55	$R_5' = 42.75$
	-	30.83	$R_6' = 43.32$
		-	

$5.297 \cdot \sqrt{56.12} = 5.297 \times 7.4$
$= 39.67$

$5.484 \times 7.49 = 41.06$

$5.612 \times 7.49 = 42.03$

$5.708 \times 7.49 = 42.75$

$5.784 \times 7.49 = 43.32$

---

(68) The first entry in the column shown above is 105.29. Since this is the ________ difference between the means D and F, designated $(D-F)'$, which are 6 steps apart, we must use R_6' to test it. We find that $105.29 > R_6' = 43.32$. Thus, this adjusted difference is statistically ____________. The adjusted difference, 105.29, is then marked with three asterisks because we have chosen the o.1% level of significance.

significant

do not

(69) The next entry, 67.00 is the adjusted difference between the means D and A, designated $(D-A)'$, which are ________ steps apart. Thus, R'_5 must be used for comparison. Since $67.00 > R'_5 = 42.75$, the adjusted difference is statistically significant. Again, 67.00 is marked with ________. You are supposed to continue the testing until an adjusted difference which does not exceed its corresponding R' is reached.	adjusted significant
(70) As we have seen in Frames 23 through 26, for the experiment with equal n's no further tests were made beyond this point, in the column. For the experiment with unequal n's, however, you must do the following, once an adjusted difference fails to exceed its corresponding R' (critical range): (1) Continue your testing (the method will be discussed later) if either of the extreme means has fewer n (number of cases) than any of the other means (if they exist) situated between the two means; (2) stop your testing if the above condition does not exist. That is, we shall have to investigate n's (numbers of cases) for the two extreme means and the means in between before we can decide whether to ________ or stop the testing, in the column.	5 three asterisks (***)
(71) For instance, we shall encounter the problem when we take the fourth entry in the column headed D. Since $31.55 \not> R'_3 = 41.08$, we must check the n's for the two extreme means D and C as well as the mean E in between. We find that $n_d = 4 < n_e = 6$. That is, condition (1) mentioned in Frame (70) exists. Thus, the testing (or comparison) must be ________ within that column.	continue

(72) At this point, it is suggested that the subset CED be written down before testing the adjusted difference (D-E)' in the next step. This will serve as a reminder that (D-E)' must be compared with R'_3 and not R'_2 as would otherwise have happened. Thus, the test sequence thus far now looks like this:

Seq. 1. $(D-F)' > R'_6$, $(D-A)' > R'_5$, $(D-B)' > R'_4$, $(D-C)' \ngtr R'_3$. CED: $(D-E)' \ngtr R'_3$.
Result $\longrightarrow$ (CED)

You may notice that the testing for the subset CED terminated after finding the next adjusted difference (D-E)' to be smaller than R'_3. In other words, the subset CED was shown to be homogeneous and should be recorded (CED) at the end of the test sequence. An important point to remember here is that the adjusted difference (D-E)', which are two steps apart, are compared with ________ but not with ________.

continued

---

(73) The next test sequence, Sequence 2, has to do with the column headed E in Table 12. Moving from top to bottom, we find the following:

Table 12-f Duncan's Test (II)

C (6) 67.34	E (6) 67.67	D (4) 31.75	Critical ranges
83.99	83.76***	106.20***	R'_2 = 39.67
41.81	42.53***	67.00***	R'_3 = 41.08
38.15	38.92	64.82***	R'_4 = 42.03
-	0.81	31.55	R'_5 = 42.75
	-	30.33	R'_6 = 43.32
		-	

Seq: 3. $(E-F)' > R'_5$, $(E-A)' > R'_4$, $(E-B)' \ngtr R'_3$. BCE: $(E-C)' \ngtr R'_3$
Result $\longrightarrow$ (BCE)

The adjusted difference (E-B)' = 38.92 was found smaller than R'_3=41.08, therefore, the subset ______ was written down and the next subset $\overline{(E-C)}'$ was further tested by ________ not ________ Since it did not exceed the critical range, the test was terminated.

(74) You are supposed to proceed to the next column headed C. Now, Test Sequence 3 follows:

Table 12-g Duncan's Test (II)

B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical range
42.93	82.99***	83.76***	105.29***	$R'_2 = 39.67$
5.80	41.81***	42.53***	67.00***	$R'_3 = 41.08$
-	38.15	38.92	64.82***	$R'_4 = 42.03$
	-	0.81	31.55	$R'_5 = 42.75$
		-	30.83	$R'_6 = 43.32$

Seq. 3. $(C-F)' > R'_4$, $(C-A)' > R'_3$, $(C-B)' \not> R'_2$.

Result ——→ (BC)

The test result shows that the subset ______ is homogeneous

BCE

R'_3

R'_2

---

(75) Next, take the column headed B. And Test Sequence 4 follows.

Table 12-h Duncan's Test (II)

A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical ranges
34.68	42.93***	82.99***	83.76***	105.29***	R'_2=39.67
-	5.80	41.81***	42.53***	67.00***	R'_3=41.08
	-	38.15	38.92	64.82***	R'_4=42.03
		-	0.81	31.55	R'_5=42.75
			-	30.83	R'_6= 43.32

Seq. 4: $(B-F)' > R'_3$, $(B-A)' \not> R'_2$.

Result ——→ (AB)

The test result shows that the subject AB is ______________.

BC

(76) Finally, the column headed A is taken and Test Sequence 5 follows:

Table 12-i Duncan's Test (II)

F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical range
-	34.68	42.93***	82.99***	83.76***	105.29***	$R'_2=39.67$
	-	5.80	41.81***	42.53***	67.00***	$R'_3=41.06$
		-	38.15	38.92	64.82***	$R'_4=42.03$
			-	0.81	31.55	$R'_5=42.75$
				-	30.83	$R'_6=43.32$
					-	

Seq. 5: $(A-F) \not> R'_2$

Result→(FA)

This means that the subset ________ is homogeneous.

homogeneous

---

(77) To conclude the multiple comparisons, the results of the test may be summarized schematically at the bottom of Table 12 as shown below, by putting homogeneous groups (that is, those treatments which do not differ significantly) in parentheses. As you may recall we ____________ the homogenous subsets (refer back to Frames 27 and 37) for Duncan's test with equal n's. This method cannot be used here because of the difference in the order of means in various homogeneous subsets which will be discussed in Frames 82-85.

Table 12-j Duncan's Test (II)

Mean	F (5) 31.80	A (4) 48.25	B (5) 51.00	C (6) 67.34	E (6) 67.67	D (4) 81.75	Critical ranges
F 31.80	-	34.88	42.93***	82.99***	83.76***	105.29***	$R'_2=39.67$
A 48.25		-	5.80	41.81***	42.53***	67.00***	$R'_3=41.08$
B 51.00			-	38.15	38.92	64.82***	$R'_4=42.03$
C 67.34				-	0.81	31.65	$R'_5=42.75$
E 67.67					-	30.83	$R'_6=43.32$
D 81.75						-	

Seq. 1:				(C	E	D)
Seq. 2:			(B	C	E)	
Seq. 4:		(A	B)			
Seq. 5:	(F	A)				

FA

(78) Any two means appearing together within the same parentheses are not significantly different. Any two means not appearing together within the same parentheses are ______ different. Thus, treatment B is not significantly different from treatments ______, ______, and ______; but significantly different from treatments ______ and ______.	underscored (or underlined)
(79) Treatment D is significantly different from Treatments ______, ______ and ______; but not significantly different from Treatments ______ and ______. So, in interpreting your test results, your intention is called to the parentheses and the subsets in and out of them.	significantly A, C, and E F and D
(80) It must be noted here that we do not put the result of the Test Sequence 3 at the bottom of the above table. In Sequence 3 (refer back to Frame 74) we found that the subset BC was homogeneous. Since BC is a subset of a larger subset BCE which has already proved to be homogeneous in Frame 73, we do not have to show BC separately from ______.	F, A, and B C and E
(81) The testing for a subset terminates either when it is shown to be homogeneous or when the subset is found to be completely included within another subset already shown to be homogeneous. For example, CED is recorded (CED) at the end of the first sequence (Frame 72) to denote its homogeneity. Sequence 3 (Frame 74) provides no additional homogeneous subsets because it terminates at subset BC which is included in ( ) already shown to be homogeneous. (Frame 73)	BCE

(82) Two more points concerning the testing procedures are in order. First, when an adjusted difference between the top mean (the largest mean) in a subset of k means and an internal mean is found to exceed R'_k, the internal mean is dropped to give the next subset to be tested. For example, suppose we have five means which are ranked and have replication numbers as follows:

P	S	R	T	Q
(3)	(2)	(5)	(5)	(3)

Suppose, further, that we have found:

$(Q-P)' > R'_5$ and $(Q-S)' \not> R'_4$

And since the two extreme means (P and Q) have fewer replications than that of R and T. SRTQ is written down and the adjusted difference $(Q-R)'$ is tested against __________ as follows (see Frame 70):

Seq.: $(Q-P)' > R'_5$, $\underline{(Q-S)' \not> R'_4}$. $\underline{SRTQ}$:

$\underline{(Q-R)' > R'_4}$

BCE

---

(83) This is the condition mentioned at the beginning of the forgoing frame. That is, the adjusted difference between the top mean (Q) in the subset (SRTQ) of 4 means and an internal mean (R) is found to exceed R'_4. Therefore the internal mean __________ is dropped to give the next subset to be tested. Thus,

Seq.: $(Q-P)' > R'_5$, $(Q-S)' \not> R'_4$. SRTQ:
$\underline{(Q-R)' > R'_4}$. $\underline{STQ}$: $(Q-S)' \not> R'_3$,
$(Q-T)' \not> R'_3$, $(T-S)' \not> R'_3$
Result —— (STQ)

R'_4

(84) Second, when an adjusted difference, not involving the top mean (the largest mean), is found to exceed R'_k, two subsets may qualify for further testing in the same dequence. For instance, suppose four means were ranked and had replication numbers as follows:

U	X	V	W
(1)	(50)	(100)	(1)

the testing steps could be

Seq.: $(W-U)' \not> R'_4$. UXVW: $(W-X)' \not> R'_4$
$(W-V)' \not> R'_4$, $(V-U)' \not> R'_4$, $(V-X)' >$
R'_4,

and the subsets UVW and UXW would both qualify for testing against ____________ in further steps in the same sequence.

R

---

(85) It is then possible for more than one homogeneous subset to be found in a single sequence. For instance, in the case of the means UXVW discussed above, the sequence could terminate with the results (UVW) and (UXW) or even (UW), (XW), and (VW) depending on the other data involved. Because of this, we put homogeneous groups in parentheses rather than ____________ them with the same lines.

R'_3

6. The Dunnett's Test for Equal n's -- with treatment means

(86) So far, we have dealt with multiple comparisons for the experiments in which no control group is involved. Very often, however, the primary interest of the researcher is to compare each of a number of different treatments with the control group in the experiment. To accomplish this one may use the Dunnett's test. Suppose we have one control condition and k treatments (excluding the control group) in our experiment. By utilizing the Dunnett's test we can compare ________ of the k treatments with the control condition.

underscoring (or underlining)

(87) Let us now consider the Dunnett's test. Suppose we have a completely randomized experiment with the following data.

each

Table 13-a Observed Data (III)

	Control	T_1	T_2	T_3	T_4
	10	18	6	17	8
	8	10	15	9	20
	6	8	9	10	4
	11	25	7	18	3
	5	18	16	20	10
	9	24	8	24	8
	8	30	11	8	5
	5	26	7	12	7
	7	15	9	14	6
	6	19	5	18	10
Sum	75	193	93	148	81
Mean	7.5	19.3	9.3	14.8	8.1

Our interest is to compare each of the four treatments with the ________.

(88) As in the previous test procedures, an analysis of variance is to be made. To do this, you may or may not code the observed data shown in Table 13-a. Let us code the data by subtracting ________ from each score and rearrange the table in such a way that coded X's and X^2's appear in the same table, we have

control (or control group)

Table 13-b X, X^2, X, & X^2 (III)

Control X	Control X^2	T_1 X	T_1 X^2	T_2 X	T_2 X^2	T_3 X	T_3 X^2	T_4 X	T_4 X^2	Total X	Total X^2
0	0	8	64	-4	16	7	49	-2	4		
-2	4	0	0	5	25	-1	1	10	100		
-4	16	-2	4	-1	1	0	0	-6	36		
1	1	15	225	-3	9	8	64	-7	49		
-5	25	8	64	6	36	10	100	0	0		
-1	1	14	196	-2	4	14	196	-2	4		
-2	4	20	400	1	1	-4	16	-5	25		
-5	25	16	256	-3	9	2	4	-3	9		
-3	9	5	25	-1	1	4	16	-4	16		
-4	16	9	81	-5	25	8	64	0	0		
Sum -25	101	93	1315	-7	127	48	510	-19	243	90	2296

---

(89) Since this is a completely randomized experiment, the total variance can be partitioned into two components: (1) the between group variation and (2) the ______________ variation which is considered to be the error variation.

10

Table 14-a Source of variation

Source of Variation	SS	df	s^2	F
Between groups Within groups				
Total				

(90) The sums of squares (SS's) are to be computed next. First of all, let us compute the correction term (refer back to Frame 88 for the necessary data):

$$\text{correction term} = \frac{(\Sigma X)^2}{N}$$

$$= \underline{\qquad\qquad\qquad}$$

$$\text{Thus,}\quad SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$

$$= \underline{\qquad\qquad\qquad}$$

$$SS_{between} = \frac{1}{n}[(\Sigma X_c)^2 + (\Sigma X_{t_1})^2 + (\Sigma X_{t_2})^2 + (\Sigma X_{t_3})^2 + (\Sigma X_{t_4})^2] - \frac{(\Sigma X)^2}{N}$$

$$= \underline{\qquad\qquad\qquad}$$

$$SS_{within} = SS_{total} - SS_{between}$$

$$= \underline{\qquad\qquad\qquad}$$

Table 14-b Sums of Squares

Source of Variation	Sum of Squares	df	s^2	F
Between G. Within G.	1036.8 1097.2			
Total	2134.0			

within-group

(91) The degrees of freedom (df's) corresponding to the respective sums of squares are determined next.

df_{total} = N-1 = ____________

$df_{between}$ = k-1 = ____________

df_{within} = df_{total} - $df_{between}$ = ________

where N represents the total number of cases and k denotes the number of groups in the experiment.

Table 14-c Degrees of Freedom

Source of Variation	Sum of Squares	Degree of freedom	s^2	F
Between G. Within G.	1036.8 1097.2	4 45		
Total	2134.0	49		

$$\frac{(90)^2}{50} = 162$$

2296 - 162 = 2134

$$\frac{1}{10}\left[(-25)^2 + (93)^2 + (-7)^2 + (48)^2 + (-19)^2\right] - 162 = 1036.8$$

2134 - 1036.8 = 1097.2

(92) Two variance estimates, one based on the between-group variation the other based on the ____________ variation, are to be computed next. The variance estimate denoted by s^2 is also called as the ____________ denoted by MS. Thus,

$$s^2_{between} = \frac{SS_b}{df_b} = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$s^2_{within} = \frac{SS_w}{df_w} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Table 14-d Variance Estimates

Source of Variation	Sum of Squares	Degree of freedom	Variance estimate	F
Between G. Within G.	1036.8 1097.2	4 45	259.20 24.38	
Total	2134.0	49		

50 - 1 = 49

5 - 1 = 4

49 - 4 = 45

(93) The observed F-ratio is, then, obtained as follows:

$$F_{ob} = \frac{s_b^2}{s_w^2} = \_\_\_\_\_\_\_\_\_\_\_\_$$

A portion of the Table F is given below. Since $df_b = 4$ and $df_w = 45$, we are supposed to check the entried in Column 4 and row 45.

Table 15 A Portion of Table of F

n_2 \ n_1	3	4	5	
40	2.84	2.61	2.45	—— d = 0.05
	4.31	3.83	3.51	—— d = 0.01
	6.60	5.70	5.13	—— d = 0.001
60	2.76	2.52	2.37	—— d = 0.05
	4.13	3.65	3.34	—— d = 0.01
	6.17	5.31	4.76	—— d = 0.001

By consulting this F table, we find that $F_{ob} = 10.63$ exceeds $F_{.95(4,45)}$, $F_{.99(4,45)}$, $F_{.999(4,45)}$. Thus the observed F-ratio is __________ beyond the 0.1% level.

within-group (or error)

mean square

$\frac{1036.8}{4} = 259.20$

$\frac{1097.2}{45} = 24.38$

---

(94) This significant F-ratio leads to reject the __________ that assumes that there is no difference between the five groups (including the control and the four treatments).

Table 14-e F-ratio

Source of variation	Sum of squares	Degree of freedom	Variance estimate	F-ratio
Between G. Within G.	1036.8 1097.2	4 45	259.20 24.38	10.63***
Total	2134.0	49		

$\frac{259.20}{24.38} = 10.63$

significant

(95) This result does not, however, tell which one of the four treatments is significantly different from the control and which one is not. To do the comparisons we may use the ______________ test.

null hypothesis

---

(96) At this point you are suggested to construct a table as shown below in which the means of the four treatments and the control are presented.

Table 16-a Dunnett's Test (III)

Mean \ Mean	Control 7.5
T_1 19.3	
T_2 9.3	
T_3 14.8	
T_4 8.1	

And then the difference between each one of the four treatments and the ______________ is entered in the table. Thus, we have

Table 16-b Dunnett's Test (III)

Diff.	Control 7.5
T_1 19.3	11.8
T_2 9.3	1.8
T_3 14.8	7.3
T_4 8.1	0.6

Dunnett's

(97) Next, the critical t value is to be determined. Since we decided in advance that the four treatments are to be compared with the control, our test of significance in this case will be of one-tailed test. Let the level of significance be $\alpha = 0.01$ and consult the Table of t for One-Sided Comparisons between k Treatments, we have

$$t = 2.92 - (2.92-2.87)\,\frac{45-40}{60-40}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

Table 17 A portion of Table of t for One-Sided Comparisons between k Treatments with $\alpha = 0.01$

df \ k	3	4	5
40	2.82	2.92	2.99
60	2.78	2.87	2.94

Control
(control group)

(98) The standard error of difference between two means, designated $s_{\bar{x}_k - \bar{x}_c}$, is to be obtained next. From Frame 92, we have $s^2_{within} = 24.38$. Thus, $s^2_{error} =$ _______________ substituting the value in the following formula, we have

$$s_{\bar{x}_k - \bar{x}_c} = \sqrt{\frac{2\,s^2_{error}}{n}} = \underline{\qquad\qquad\qquad}$$

2.91

[also $2.87 + (2.92-2.87)\,\dfrac{60-45}{60-40} = 2.91$]

(99) Now, we can obtain the critical range, designated R, by the following formula.

$$R = t \cdot s_{\bar{x}_k - \bar{x}_c} = \underline{\hspace{6cm}}$$

This value is then entered in Table 16 as shown below.

Table 16-c Dunnett's Test (III)

Mean	Control 7.5	Critical range
T_1 19.3	11.8**	R = 6.4
T_2 9.3	1.8	
T_3 14.8	7.3**	
T_4 8.1	0.6	

$s^2_{within} = 24.38$

$$\sqrt{\frac{2 \times 24.38}{10}} = 2.2$$

---

(100) Finally, the differences are compared with the critical range, R. Those differences which exceed R= 6.4 are ______________ at the 1% (or $\alpha = 0.01$) level, and are marked with two asterisks (**). In conclusion, the means of ___________ and ______________ are significantly greater than the mean of the ___________; the remaining treatment means are not. With probability of ______________ that these statements are correct.

2.91 x 2.2 = 6.4

---

significant

T_1 and T_3

control

0.99

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June, 1965

CORRELATION

(Programmed Instruction)

by

Heng-shu Tseng

Institute of Educational Research

Indiana University

TABLE OF CONTENTS

Page

I. Introduction (Frames 1 - 12) 1

II. Pearson's Product-Moment Coefficient of Correlation (Frames 13 - 51) 7

  1. Raw Score Method - Ungrouped Data (Frames 19 -26) 10

  2. Coded Score Method - Ungrouped Data (Frames 27 - 30) 14

  3. Deviation Method - Ungrouped Data (Frames 31 - 35) 17

  4. Standard Score Method - Ungrouped Data (Frames 36 - 40) 20

  5. Scatter Diagram Method - Grouped Data (Frames 41 - 51) 24

III. Biserial Coefficient of Correlation (Frames 52 -65) 34

I. INTRODUCTION

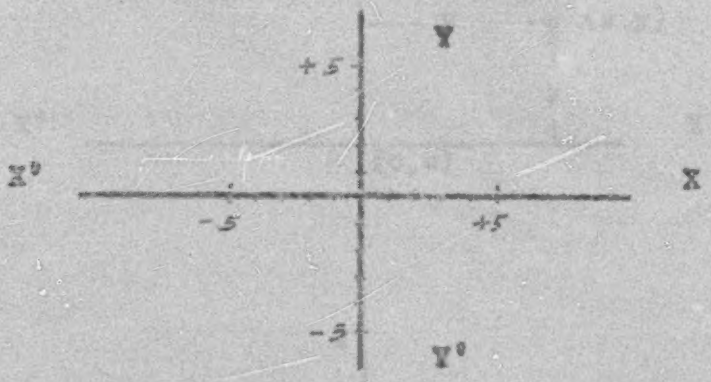
(1) In dealing with the continuous variables, the primary interest of the researcher is very often centered around the analysis of interrelations of these variables. Suppose that we have two sets of data representing two ______________ variables X and Y which are paired, the correlation can be determined by noting how much of a change in Variable X is associated with change in ______________

(2) The correlation between the two variables can be graphically represented by plotting the paired points in the Cartesian co-ordinate system which is composed of a horizontal axis XX' and a vertical axis YY', as shown below.

continuous

Variable Y

Fig. 1 Cartesian Co-ordinates



The two axis are ______________ to each other. That is,

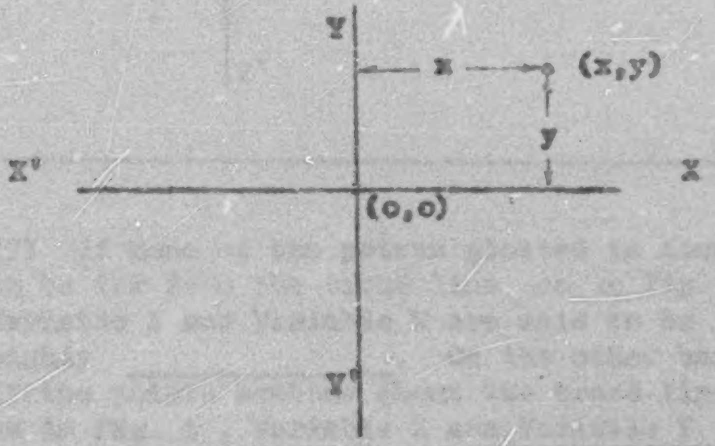
$\overline{XX'} \perp \overline{YY'}$

(3) Any point which falls to the right of the vertical axis YY' has a positive x value and any point falling to the left of the YY' axis has a ______________ x value. Similarly, any point which falls above the horizontal axis XX' has a positive ______________ value and any point falling below the XX' axis has a ______________ y value.

perpendicular

(4) The point where the two perpendicular axes, XX' and YY', cross is called the origin. Any point in the Cartesian co-ordinate system may be indicated by quoting its x-coordinate (distance from $\overline{YY'}$) and y-coordinate (distance from $\overline{XX'}$) in the form of (x,y). Thus, the origin can be indicated by (o,o). The horizontal co-ordinate (or x-coordinate) of a point is called the abscissa and its vertical co-ordinate (or y-coordinate) is called the ordinate. Indicate a point, P, whose abscissa is + 4 and ordinate is + 3; another point, Q, whose abscissa is - 3 and ordinate is + 2.

Fig. 2 Origin, Abscissa, & Ordinate

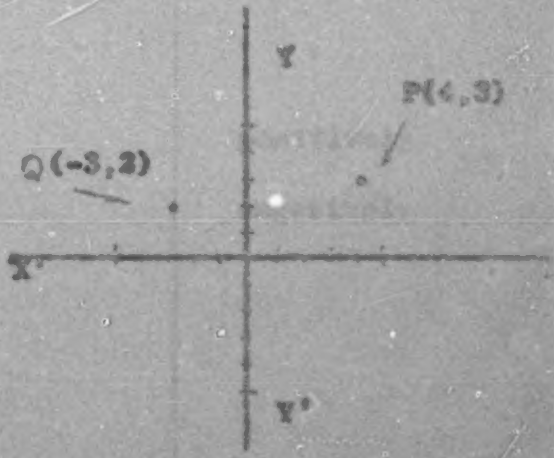


(5) Now, suppose we have the paired data of Variable X and Variable Y plotted and we find that the points show a trend, we would say that Variable X and Variable Y are ______________. This is equivalent to saying that we can make some sort of prediction of the value of y (or x), given a knowledge of the value of x (or y).

negative

y

negative



(6) If an increase in x results in an increase in y, as shown in Fig. 3 below, Variabel X and Variable Y are said to be ____________ correlated. If, as shown in Fig. 4 below, increasing x is associated with decreasing y, Variable X and Variable Y are said to be ____________ correlated.

Fig. 3 Positive Correlation

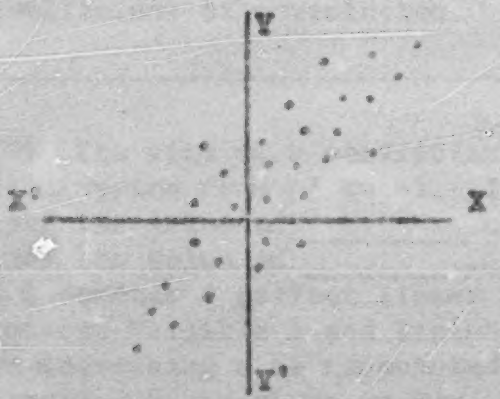
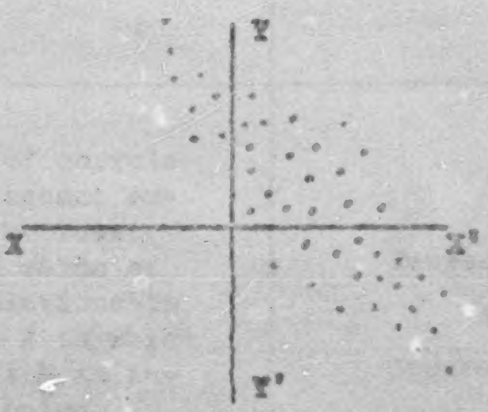


Fig. 4 Negative Correlation



correlated

(7) If none of the points plotted is found to be far from the trend line, as in Fig. 5, Variable X and Variable Y are said to be highly ____________. On the other hand, if the points scatter about the trend line, as in Fig. 6 , Variable X and Variable Y are said to be in ____________ correlation.

Fig. 5 High Correlation

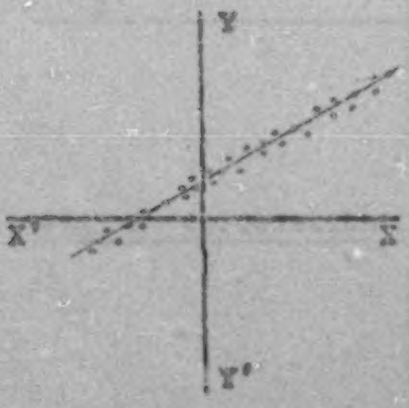
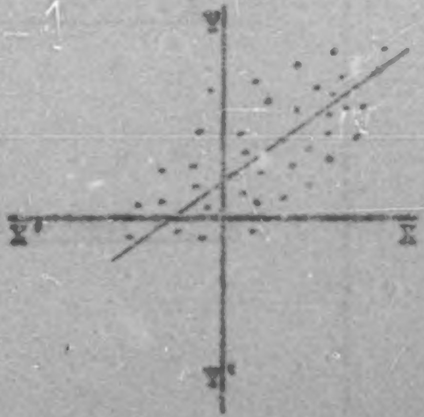


Fig. 6 Low Correlation



positively

negatively

(8) The degree of correlation is measured by the coefficient of correlation which is defined as

$$r = \frac{\Sigma x\ y}{N\ \sigma_x\ \sigma_y}$$ by Karl Pearson,

and as $$\rho = 1 - \frac{6\ \Sigma\ D^2}{N(N^2-1)}$$ by Charles Spearman.

These are two widely used coefficients of correlation which will be discussed later in the program. The symbol r denotes the ______________ product-moment coefficient of correlation and the symbol ρ (reads rho) represents the ______________ rank order coefficient of correlation.

correlated

low

---

(9) The size of a coefficient of correlation varies from +1 to -1. It cannot exceed the value of ________ or be less than the value of ________. A value of +1 denotes a perfect linear relationship between Variable X and Variable Y in which an increasing x is associated with an increasing y, as shown in Fig. 7 below. When the coefficient of correlation is equal to - 1, Variable X and Variable Y again have a perfect ________________ relationship in which an increasing x is associated with a decreasing y, as shown in Fig. 8.

Pearson's

Spearman's

Fig. 7 Correlation Coefficient = + 1

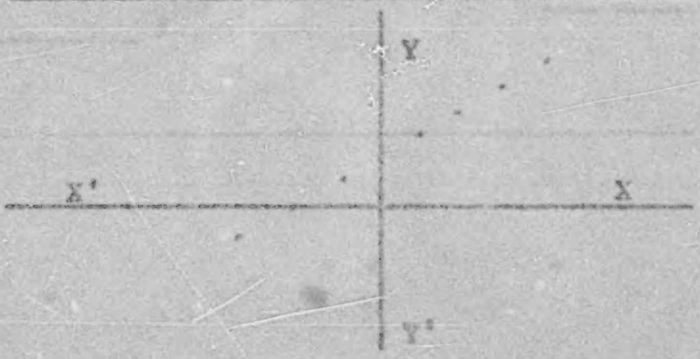
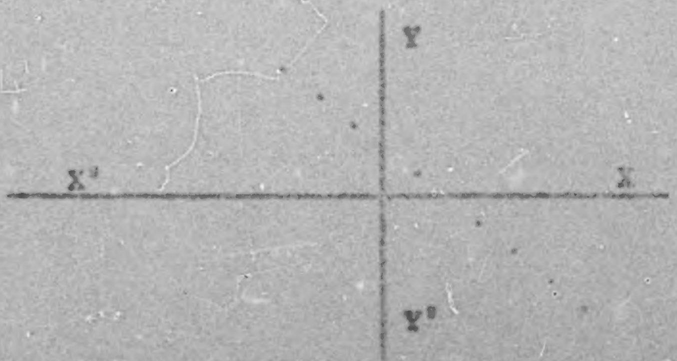
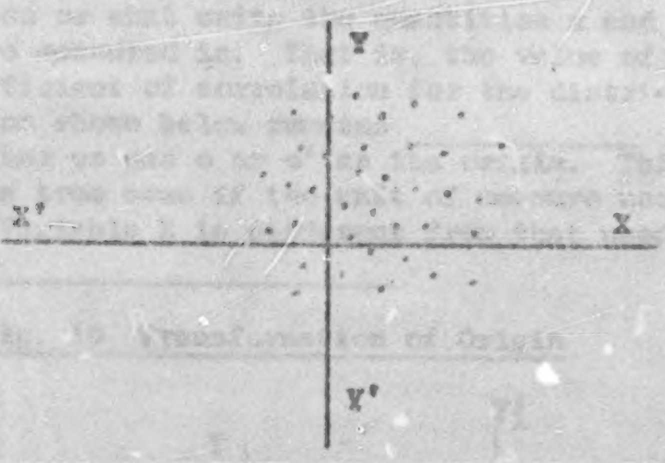


Fig. 8 Correlation Coefficient = - 1



(10) When the coefficient of correlation is equal to zero, there is no __________ relation existing between x and y, as is shown in Fig. 9 below

Fig. 9 Coefficient of Correlation = 0



+ 1

- 1

linear

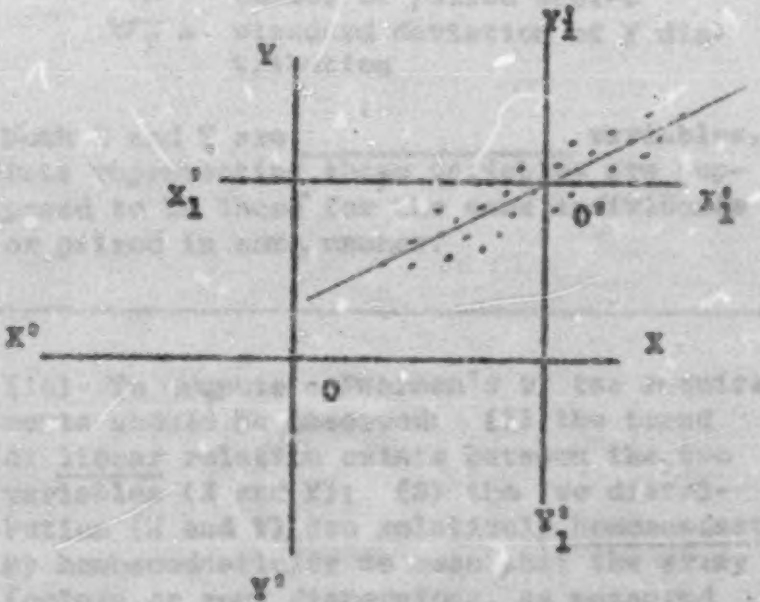
---

(11) The values of coefficient of correlation other than + 1, 0, and - 1 indicate that while there is no strictly ______________ relationship between Variable X and Variable Y, there is a trend. A positive coefficient indicates that there is a ______________ for increasing x to be associated with __________________ y. A negative coefficient shows that increasing x tends to be associated with _______________________ y.

linear

(12) It must be mentioned that the value of a coefficient of correlation remains the same (constant) for a given distribution (paired points for Variable X and Variable Y) regardless of where the origin of the Cartesian co-ordinate system is chosen or what units the quantities x and y are measured in. That is, the value of coefficient of correlation for the distribution shown below remains ____________ whether we use o or o' as the origin. This holds true even if the unit of measure used for Variable X is different from that used for ______________.

Fig. 10 Transformation of Origin



linear

trend

increasing

decreasing

XI. Pearson's Product-Moment Coefficient of Correlation

(13) The Pearson's product-moment coefficient of correlation is the commonly computed coefficient which is defined by

$$r = \frac{\Sigma xy}{N \sigma_x \sigma_y}$$

where r = coefficient of correlation
x = deviation from mean of Variable X, $(X - \overline{X})$
y = deviation from mean of Variable Y, $(Y - \overline{Y})$
σ_x = standard deviation of X distribution
N = Number of paired scores
σ_y = standard deviation of Y distribution

Both X and Y are ____________ variables. Data representing these variables are supposed to be those for the same individuals or paired in some manner.

the same (or constant)

Variable Y

---

(14) To compute a Pearson's r, two requirements should be observed: (1) the trend of linear relation exists between the two variables (X and Y); (2) the two distribution (X and Y) are relatively homoscedastic. By homoscedasticity we mean that the array (column or row) dispersions, as measured by the array standard deviations, are approximately equal for all arrays (columns or rows) in the correlation diagram (that is, the Cartesian Co-ordinate system). Generally, homoscedasticity prevails when the two distributions are fairly symmetrical and unimodal, not necessarily normal, within themselves. Thus, before computing a Pearson's ________ coefficient of correlation, r, one may (1) plot the paired points in the __________ co-ordinate system to see whether linear relation exists, and (2) plot two separate distributions (X and Y) to see whether __________ prevails.

continuous

(15) Now, let us consider an example in which a total of 20 subjects have been observed on the basis of Variable X and Variable Y and resulted with the following data:

Table 1 Observed Data (I)

Subject	X	Y	Subject	X	Y
1	57	142	11	86	181
2	69	148	12	93	190
3	71	150	13	71	152
4	60	147	14	84	178
5	80	172	15	69	155
6	87	184	16	79	170
7	61	150	17	81	175
8	65	153	18	73	160
9	46	130	19	74	162
10	77	165	20	76	164

Before computing a product-moment coefficient of correlation, we should check for (1) linear relation between the two distributions (X and Y) and (2) ______________ for both distributions.

product-moment

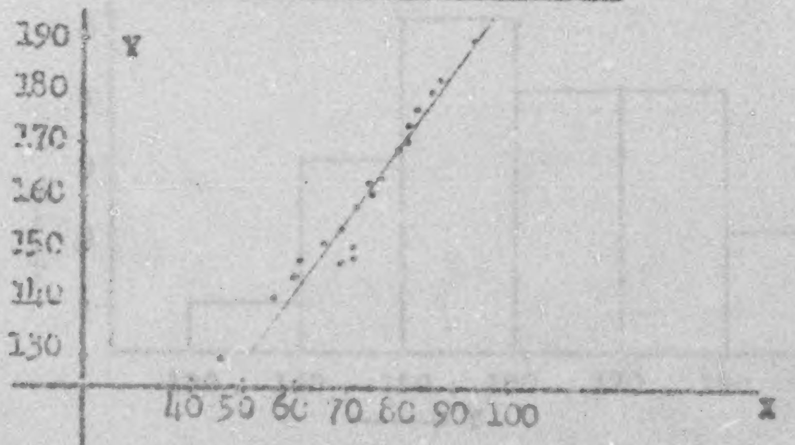
Cartesian

homoscedasticity

---

(16) To see whether the X and Y distributions have the trend of linear relationship, you are supposed to use the Cartesian ________________ system. The paired scores in Table 1 (Frame 15) are plotted in the following. This is called the correlation chart.

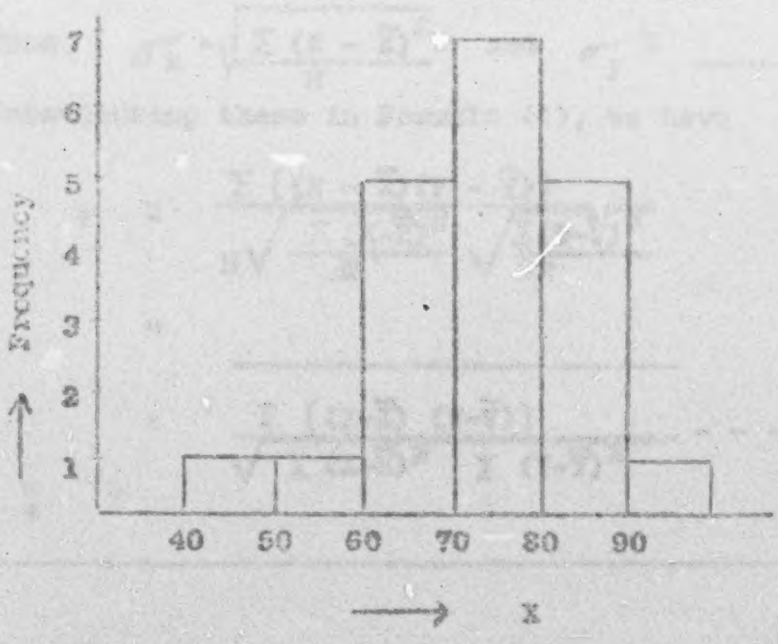
Fig. 11 Correlation Chart (I)



homoscedasticity

(17) Next, homoscedasticity may be investigated as follows: Take X variable first. Construct a histogram (which is a frequency distribution in which rectangles replace the plotted points) in which the horizontal axis represents X and the vertical axis represents frequency, as shown below.

Fig. 12 Histogram for Variable X (I)

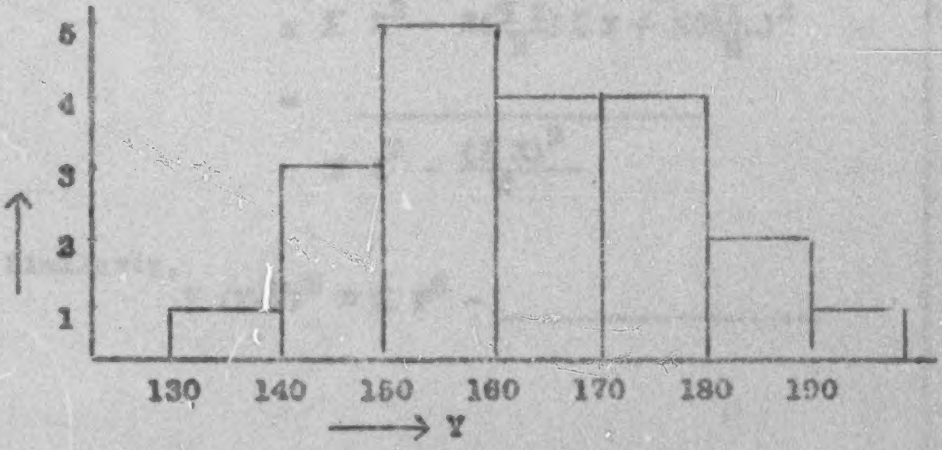


This distribution is unimodal and rather symmetrical, and is relatively ____________.

co-ordinate

(18) Variable Y is taken, next. Its histogram is shown below.

Fig. 13 Histogram for Variable Y (I)



This distribution, too, seems to meet the requirement of homoscedasticity. Thus, we are justified to compute the ____________ coefficient of correlation.

homoscedastic

(19) The Raw Score Method (Ungrouped Data)
By definition (see Frame 13), the product-moment r is given by

$$r = \frac{\Sigma xy}{N \sigma_x \sigma_y} \text{ - - - - - } (1)$$

Let X and Y denote raw scores. Then $x = (X - \bar{X})$ and $y = (Y - \bar{Y})$ where $\bar{X}$ and $\bar{Y}$ represent the means of the two distributions. By definition, $\sigma_x = \sqrt{\frac{\Sigma x^2}{N}}$ and $\sigma_y = \sqrt{\frac{\Sigma y^2}{N}}$

Thus, $\sigma_x = \sqrt{\frac{\Sigma (X - \bar{X})^2}{N}}$ and $\sigma_y =$ _______.

Substituting these in Formula (1), we have

$$r = \frac{\Sigma [(X - \bar{X})(Y - \bar{Y})]}{N\sqrt{\frac{\Sigma (X-\bar{X})^2}{N}} \sqrt{\frac{\Sigma (Y-\bar{Y})^2}{N}}}$$

$$= \frac{\qquad\qquad}{\qquad\qquad}$$

$$= \frac{\Sigma [(X-\bar{X})(Y-\bar{Y})]}{\sqrt{\Sigma (X-\bar{X})^2 \; \Sigma (Y-\bar{Y})^2}} \text{ - - - } (2)$$

Pearson's product-moment

(20) Notice that $\Sigma (X-\bar{X})^2$ or $\Sigma (Y-\bar{Y})^2$ in Formula (2) shown above is usually referred to as the sum of squares. By definition,

$$\bar{X} = \frac{\Sigma X}{N} \text{ and } \bar{Y} = \underline{\qquad\qquad}.$$

Thus,
$$\Sigma (X-\bar{X})^2 = \Sigma (X^2 - 2\bar{X}X + \bar{X}^2)$$
$$= \Sigma X^2 - 2\bar{X}\Sigma X + \Sigma \bar{X}^2$$
$$= \Sigma X^2 - 2\bar{X}\Sigma X + N\bar{X}^2$$
$$= \Sigma X^2 - 2\left(\frac{\Sigma X}{N}\right)\Sigma X + N\left(\frac{\Sigma X}{N}\right)^2$$
$$= \underline{\qquad\qquad\qquad}$$
$$= \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$

Similarly,
$$\Sigma (Y-\bar{Y})^2 = \Sigma Y^2 - \underline{\qquad\qquad\qquad}$$

$$\sqrt{\frac{\Sigma (Y-\bar{Y})^2}{N}}$$

$$\frac{\Sigma [(X-\bar{X})(Y-\bar{Y})]}{N\sqrt{\frac{\Sigma (X-\bar{X})^2 \; \Sigma (Y-\bar{Y})^2}{N}}}$$

(21) In the same manner, the means $\bar{X}$ and $\bar{Y}$ in $\Sigma[(X-\bar{X})(Y-\bar{Y})]$ can also be eliminated as follows:

$$\begin{aligned}\Sigma[(X-\bar{X})(Y-\bar{Y})] &= \Sigma(XY-\bar{X}Y-\bar{Y}X+\bar{X}\bar{Y})\\ &= \Sigma XY-\bar{X}\Sigma Y-\bar{Y}\Sigma X+\Sigma\bar{X}\bar{Y}\\ &= \Sigma XY-\left(\frac{\Sigma X}{N}\right)\Sigma Y-\left(\frac{\Sigma Y}{N}\right)\Sigma X+N\frac{\Sigma X}{N}\frac{\Sigma Y}{N}\\ &= \underline{\hspace{6cm}}\\ &= \Sigma XY-\frac{(\Sigma X)(\Sigma Y)}{N}\end{aligned}$$

$$\frac{\Sigma Y}{N}$$

$$\Sigma X^2-2\frac{(\Sigma X)^2}{N}+\frac{(\Sigma X)^2}{N}$$

$$\frac{(\Sigma Y)^2}{N}$$

---

(22) Now, it should become clear that we may be able to compute the product-moment r with nothing but basically two sets of raw scores, that is, X's and Y's. Substituting $\Sigma(X-\bar{X})^2=\Sigma X^2-\frac{(\Sigma X)^2}{N}$, $\Sigma(Y-\bar{Y})^2=\Sigma Y^2-\frac{(\Sigma Y)^2}{N}$, and $\Sigma[(X-\bar{X})(Y-\bar{Y})]=\Sigma XY-\frac{(\Sigma X)(\Sigma Y)}{N}$ in Formula (2), Frame 19, we have

$$\begin{aligned}r &= \frac{\Sigma[(X-\bar{X})(Y-\bar{Y})]}{\sqrt{\Sigma(X-\bar{X})^2\,\Sigma(Y-\bar{Y})^2}}\\ &= \frac{\Sigma XY-\frac{(\Sigma X)(\Sigma Y)}{N}}{\sqrt{\left[\Sigma X^2-\frac{(\Sigma X)^2}{N}\right]\left[\Sigma Y^2-\frac{(\Sigma Y)^2}{N}\right]}}\\ &= \underline{\hspace{6cm}}\\ &= \frac{N\Sigma XY-\Sigma X\Sigma Y}{\sqrt{[N\Sigma X^2-(\Sigma X)^2][N\Sigma Y^2-(\Sigma Y)^2]}}\ \text{- - - (3)}\end{aligned}$$

$$\Sigma XY-\frac{(\Sigma X)(\Sigma Y)}{N}-\frac{(\Sigma X)(\Sigma Y)}{N}+\frac{(\Sigma X)(\Sigma Y)}{N}$$

---

(23) Formula (3) shown above is the formula that can be used in computing the product-moment r. From this equation, we know that we should have information on N, ΣX, ΣY, ______, ______, and ______ before we can compute the correlation coefficient.

$$\frac{N\Sigma XY-\frac{N(\Sigma X)(\Sigma Y)}{N}}{\sqrt{\left[N\Sigma X^2-\frac{N(\Sigma X)^2}{N}\right]\left[N\Sigma Y^2-\frac{N(\Sigma Y)^2}{N}\right]}}$$

(24) Let us see how we can go about this. You may recall that in Frame 15 an example in which X's and Y's on a total of 20 subjects was given. To get ΣX and ΣY we just add the X and Y columns as shown below:

Table 2-a Pearson's r - Raw Scores

Subject	X	Y
1	57	142
2	69	148
3	71	150
4	60	147
5	80	172
6	87	184
7	61	150
8	65	152
9	46	130
10	77	165
11	85	181
12	93	190
13	71	162
14	84	178
15	69	155
16	79	170
17	81	175
18	73	160
19	74	162
20	76	164
Sum	1458	3237

Thus, we have N = ______, ΣX = _____, and ΣY = ______________.

ΣXY, ΣX^2, and ΣY^2

(25) Next, as shown in the following table three new columns headed XY, X^2, and Y^2 are added to the right of Column Y. The entries of Column XY are obtained by multiplying X's by their corresponding Y's for all the subjects. Thus, the first entry XY = 8094 is obtained by multiplying X = ________ by Y = ________. To obtain the X^2 entry for a particular subject, the corresponding X is multiplied by itself. Thus, the first entry X^2 = 3249 is obtained by multiplying X = ________ by itself. Similarly, Y^2 = 20164 = ________ x ________

Table 2-b Pearson's r - Raw Scores

Subject	X	Y	XY	X^2	Y^2
1	57	142	8094	3249	20164
2	69	148	10212	4761	21904
3	71	150	10650	5041	22500
4	60	147	8820	3600	21609
5	80	172	13760	6400	29584
6	87	184	16008	7569	33856
7	61	150	9150	3721	22500
8	65	152	9880	4225	23104
9	46	130	5980	2116	16900
10	77	165	12705	5929	27225
11	85	181	15385	7225	32761
12	93	190	17670	8649	36100
13	71	152	10792	5041	23104
14	84	178	14952	7056	31684
15	69	155	10695	4761	24025
16	79	170	13430	6241	28900
17	81	175	14175	6561	30625
18	73	160	11680	5329	25600
19	74	162	11988	5476	26244
20	76	164	12464	5776	26896
Sum	1458	3227	236490	108726	826285

(26) What we really wanted to have from the above table was the values of ΣXY, ΣX^2, and ΣY^2. By adding the respective columns, we have $\Sigma XY = 238490$, $\Sigma X^2 = 108726$, and $\Sigma Y^2 = 525285$. Substituting these and values of N, ΣX, and ΣY (refer back to Frame 24) in Formula 3 (refer back to Frame 22), we have the Pearson's ______________ coefficient of correlation

$$r = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2]\,[N\Sigma Y^2 - (\Sigma Y)^2]}}$$

$$= \overline{\qquad\qquad\qquad\qquad}$$

$$= \frac{4769800 - 4704966}{\sqrt{(2174520-2125764)(10505700-10413529)}}$$

$$= \frac{64834}{\sqrt{4493889276}}$$

$$= \overline{\qquad\qquad\qquad\qquad}$$

$$= 0.97$$

57 by 142

57

142 x 142

---

(27) The Coded Score Method (Ungrouped Data) The calculation of the product-moment r can be simplified considerably by the coded score method in which the raw scores X's and Y's are coded by subtracting a constant a from X's and another constant b from Y's. Notice that a = b or a ≠ b. This means that the two constants need not be ______________. Replacing X, and Y, with X-a and Y-b in Formula (3):

$$r = \frac{N\Sigma XY - \Sigma X \Sigma Y}{\sqrt{[N\Sigma X^2-(\Sigma X)^2][N\Sigma Y^2-(\Sigma Y)^2]}},$$ we

have:

$$r = \frac{N\Sigma[(X-a)(Y-b)] - \Sigma(X-a)\,\Sigma(Y-b)}{\sqrt{\{N\Sigma(X-a)^2-[\Sigma(x-a)]^2\}\{N\Sigma(Y-b)^2-[\Sigma(Y-b)]^2\}}} \quad - - - - -(4)$$

This is the equation we shall use later.

product-moment

$$\frac{20x238490-1458x3227}{\sqrt{(20x108726-1458^2)\ (20x}}$$ cont'd

$$\frac{----}{525285-3227^2)}$$

$$\frac{64834}{67036}$$

(28) Let us now code the X's and Y's of the observed data (refer back to Frame 15). The coded scores will be (X-a)'s and (X-b)'s. The constant a or b is usually a multiple of ten equal to or smaller than the smallest raw score in the distribution. Since the smallest score in the X distribution is 46, let us select a = ______________. And we may select b = ______________ since the smallest score in the Y distribution is 130. Now we have our coded scores as follows:

Table 3-a Pearson's r--Coded Scores

Subject	X	Y	X-40	Y-130
1	57	142	17	12
2	69	148	29	18
3	71	150	31	20
4	60	147	20	17
5	80	172	40	42
6	87	184	47	54
7	61	150	21	20
8	65	152	25	22
9	46	130	6	0
10	77	165	37	35
11	85	181	45	51
12	93	190	53	60
13	71	152	31	22
14	84	178	44	48
15	69	155	29	25
16	79	170	39	40
17	81	175	41	45
18	73	160	33	30
19	74	162	34	32
20	76	164	36	34
Sum			658	627

From the table, we have $\sum(X-40)$ = __________ and $\sum(Y-130)$ = ______________.

Equal (or the same)

(29) According to equation 4 (Frame 27), besides N, $\Sigma(X-40)$, and $\Sigma(Y-130)$ we still need the values of $\Sigma[(X-40)(Y-130)]$, ________, and ____________. To obtain these values three new columns headed (X-40)(Y-130), $(X-40)^2$, and $(Y-130)^2$ are created and each column totaled as shown in the following table:

Table 3-b Pearson's r - Coded Scores

Subject	X	Y	X-40	Y-130	(X-40)(Y-130)	$(X-40)^2$	$(Y-130)^2$
1	57	142	117	12	204	289	144
2	69	148	29	18	522	841	324
3	71	150	31	20	620	961	400
4	60	147	20	17	340	400	289
5	80	172	40	42	1680	1600	1764
6	87	184	47	54	2538	2209	2916
7	61	150	21	20	420	441	400
8	65	152	25	22	550	625	484
9	46	130	6	0	0	36	0
10	77	165	37	35	1295	1369	1225
11	85	181	45	51	2295	2025	2601
12	93	190	53	60	3180	2809	3600
13	71	152	31	22	682	961	484
14	84	178	44	48	2112	1936	2304
15	69	155	29	25	725	841	625
16	79	170	39	40	1560	1521	1600
17	81	175	41	45	1845	1681	2025
18	73	160	33	30	990	1089	900
19	74	162	34	32	1088	1156	1024
20	76	164	36	34	1224	1296	1156
Sum			653	627	23870	24086	24265

The entry in Column (X-40)(Y-130) for subject 2 is 522. This values is obtained by multiplying 29 by 18. For the same subject, the entry $(X-40)^2 = 841$ is obtained by ________ x ________; and the entry $(Y-130)^2 = 324$ by ________ x ________. From this table, we have

$\Sigma[(X-40)(Y-130)] =$ ____________, $\Sigma(X-40)^2 =$

____________ and $\Sigma(Y-130)^2 =$ ____________.

40

130

653

627

(30) Substituting the necessary values in Formula (4), we have

$$r = \frac{N\Sigma[(X-40)(Y-130)] - \Sigma(X-40)\,\Sigma(Y-130)}{\sqrt{\{N\Sigma(X-40)^2-[\Sigma(X-40)]^3\}\{N\Sigma(Y-130)^2-[\Sigma(Y-130)]^2\}}}$$

$$= \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$= \frac{477400-412566}{\sqrt{(481720-432964)(485300-393129)}}$$

$$= \frac{64834}{\sqrt{4493889276}}$$

$$= \underline{\qquad\qquad\qquad\qquad\qquad}$$

$$= 0.97$$

$\Sigma(X-40)^3$

$\Sigma(Y-130)^2$

29×29

18×18

23870

24086

24265

---

(31) The Deviation Method (Ungrouped Data) As you may recall (refer back to Frame 19), the product-moment coefficient of correlation is defined as

$$r = \frac{\Sigma xy}{N\sigma_x\sigma_y}.$$

We also know that the standard deviation is defined as $\sigma_x = \sqrt{\frac{\Sigma x^2}{N}}$ or $\sigma_y = \sqrt{\frac{\Sigma y^2}{N}}$.

Thus, we may derive a computational formula, in which nothing but deviations x's, y's, x^2's, y^2's appear, as follows:

$$r = \frac{\Sigma xy}{N\sigma_x\sigma_y}$$

$$= \underline{\qquad\qquad\qquad}$$

$$= \frac{\Sigma xy}{N\sqrt{\frac{\Sigma x^2\,\Sigma y^2}{N^2}}}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

$$= \frac{\Sigma xy}{\sqrt{\Sigma x^2\,\Sigma y^2}} \text{ - - - - - - - (5)}$$

$$\frac{20\times23870-658\times627}{\sqrt{(20\times24086-658^2)(20\times24265}}$$

$$\frac{\qquad\qquad}{-627^2)}$$

$$\frac{64834}{67036}$$

(32) By definition, the deviation x is given by $X-\bar{X}$ where X denotes the raw score and $\bar{X}$ represents the mean of the distribution. Similarly y = ________________. To compute the Pearson's r with the deviation method you should construct a table as follows:

Table 4-a Pearson's r - Deviation Method

Subject	X	Y	x	y	xy	x^2	y^2
1	57	142	-15.9	-19.35			
2	69	148	- 3.9	-13.35			
3	71	150	- 1.9	-11.35			
4	60	147	-12.9	-14.35			
5	80	172	7.1	10.65			
6	87	184	14.1	22.65			
7	61	150	-11.9	-11.35			
8	65	152	- 7.9	- 9.35			
9	46	130	-26.9	-31.35			
10	77	165	4.1	3.65			
11.	85	181	12.1	19.65			
12	93	190	20.1	28.65			
13	71	152	- 1.9	- 9.35			
14	84	178	11.1	16.65			
15	69	155	- 3.9	- 6.35			
16	79	170	6.1	8.65			
17	81	175	8.1	13.65			
18	73	160	0.1	- 1.35			
19	74	162	1.1	0.65			
20	76	164	3.1	2.65			
Sum	1458	3227	0	0			
Mean	72.9	161.35					

From this table, we have $\bar{X}$ = __________ and $\bar{Y}$ = ____________. Thus, x = -15.9 for subject 1 is obtained by __________ - __________; y = -19.35 for the same subject is obtained by ____________ - ________________.

$$\frac{\Sigma xy}{N\sqrt{\frac{\Sigma x^2}{N}}\sqrt{\frac{\Sigma y^2}{N}}}$$

$$\frac{\Sigma xy}{N\sqrt{\frac{\Sigma x^2\Sigma y^2}{N}}}$$

(33) Notice that $\sum x = 0$ and $\sum y = 0$. In other words the sum of deviations for any distribution is equal to zero. This important feature may be proved as follows:

$$\begin{aligned}\sum x &= \sum (X-\bar{X}) \\ &= \sum X - \sum \bar{X} \\ &= \sum X - \underline{\qquad\qquad} \\ &= \sum X - N\frac{\sum X}{N} \\ &= \sum X - \sum X \\ &= \underline{\qquad\qquad}\end{aligned}$$

$Y-\bar{Y}$

72.9

161.35

57-72.9

142-161.35

---

(34) According to Formula 5 (Frame 31), we need the values of ________, ________, and ________. Thus, columns headed xy, x^2, and y^2 as shown below should be formulated.

Table 4-b Pearson's r-Deviation Method

Subject	X	Y	x	y	xy	x^2	y^2
1	57	142	-15.9	-19.35	307.67	252.81	374.42
2	69	148	- 3.9	-13.35	52.07	15.21	178.22
3	71	150	- 1.9	-11.35	21.57	3.61	128.82
4	60	147	-12.9	-14.35	185.12	166.41	205.92
5	80	172	7.1	10.65	75.62	50.41	113.42
6	87	184	14.1	22.65	319.37	198.81	513.02
7	61	150	-11.9	-11.35	135.07	141.61	128.82
8	65	152	- 7.9	- 9.35	73.87	62.41	87.42
9	46	130	-26.9	-31.35	843.32	723.61	982.82
10	77	165	4.1	3.65	14.97	16.81	13.32
11	85	181	12.1	19.65	237.77	146.41	386.12
12	93	190	20.1	28.65	575.87	404.01	820.82
13	71	152	- 1.9	- 9.35	17.77	3.61	87.42
14	84	178	11.1	16.65	184.82	123.21	277.22
15	69	155	- 3.9	- 6.35	24.77	15.21	40.32
16	79	170	6.1	8.65	52.77	37.21	74.82
17	81	175	8.1	13.65	110.57	65.61	186.32
18	73	160	0.1	- 1.35	0.14	0.01	1.82
19	74	162	1.1	0.65	0.72	1.21	0.42
20	76	164	3.1	2.65	8.22	9.61	7.02
Sum	1458	3227	0	0	3242.07	2437.80	4608.50
Mean	72.9	161.35					

$N\bar{X}$

0

(35) From Table 4-b, we obtain Σxy = __________, Σx^2 = __________, and Σy^2 = __________. Substituting these values in Formula (5), we have

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

= __________

$$= \frac{3242.07}{\sqrt{11234601.3}}$$

= __________

= __________

Σxy, Σx^2, and Σy^2

---

(36) The Standard Score Method (Ungrouped Data) Sometimes both the X and Y distributions (called bivariate distributions) are expressed in standard scores. That is, the raw scores X's and Y's are transformed into $\frac{x}{\sigma_x}$ and $\frac{y}{\sigma_y}$ where x and y denote deviations and σ_x and σ_y represent standard deviations in the X and Y distributions. Let us make the transformation with our present example (Table 4-b, Frame 34). In Table 4-b we have x's and y's for all the 20 subjects. To obtain standard scores for these subjects we need to compute σ_x and σ_y. By definitions $\sigma_x = \sqrt{\quad}$ __________ and $\sigma_y = \sqrt{\quad}$ __________. Thus,

σ_x = __________

σ_y = __________

3242.07

2437.80

4608.50

$$\frac{3242.07}{\sqrt{2437.80x4608.50}}$$

$$\frac{3242.07}{3351.8}$$

0.97

(37) As shown in the following table, the ________ $\frac{x}{\sigma_x}$ and $\frac{y}{\sigma_y}$ are obtained by dividing each value in Column x by $\sigma_x = 11.04$ and each entry in Column y by $\sigma_y = 15.18$. Taking the first entry of the $\frac{x}{\sigma_x}$ column, -1.44, for example:

-1.44 = ________ ÷ ________.

The first entry of the $\frac{y}{\sigma_y}$ is obtained as follows:

-1.27 = ________ ÷ ________.

Table 5-a Pearson's r-Standard Score Method

Subject	X	Y	x	y	$\frac{x}{\sigma_x}$	$\frac{y}{\sigma_y}$
1	57	142	-15.9	-19.35	-1.44	-1.27
2	69	148	- 3.9	-13.35	-0.35	-0.88
3	71	150	- 1.9	-11.35	-0.17	-0.75
4	60	147	-12.9	-14.35	-1.17	-0.95
6	87	184	14.1	22.65	1.28	1.49
5	80	172	7.1	10.65	0.64	0.70
7	61	150	-11.9	-11.35	-1.08	-0.75
8	65	152	- 7.9	- 9.35	-0.72	-0.62
9	46	130	-26.9	-31.35	-2.44	-2.07
10	77	165	4.1	3.65	0.37	0.24
11	85	181	12.1	19.65	1.10	1.29
12	93	190	20.1	28.65	1.82	1.89
13	71	152	- 1.9	- 9.35	-0.17	-0.62
14	84	178	11.1	16.65	1.01	1.10
15	69	156	- 3.9	- 6.35	-0.35	-0.42
16	79	170	6.1	8.65	0.55	0.57
17	81	175	8.1	13.65	0.73	0.90
18	73	160	0.1	- 1.35	0.01	-0.09
19	74	162	1.1	0.65	0.10	0.04
20	76	164	3.1	2.65	0.28	0.17

$\frac{\Sigma x^2}{N}$

$\frac{\Sigma y^2}{N}$

$\sqrt{\frac{2437.8}{20}} = \sqrt{121.89} = 11.04$

$\sqrt{\frac{4608.5}{20}} = \sqrt{230.43} = 15.19$

(38) Knowing the standard scores of the bivariate distributions X and Y, we may compute the Pearson's product-moment r with the following formula:

$$r = \frac{\sum[(\frac{x}{\sigma_x})(\frac{y}{\sigma_y})]}{N} \quad . . . . (6)$$

This formula is derived as follows: by definition $r = \frac{\Sigma xy}{N\sigma_x \sigma_y}$. Thus,

$$r = \frac{\Sigma xy}{N\sigma_x \sigma_y}$$

$$= \frac{1}{N}\left(\frac{\Sigma xy}{\sigma_x \sigma_y}\right)$$

$$= \frac{1}{N}\Sigma\left(\frac{xy}{\sigma_x \sigma_y}\right)$$

$$= \frac{1}{N}\ \underline{\qquad\qquad\qquad}$$

$$= \frac{\Sigma[(\frac{x}{\sigma_x})(\frac{y}{\sigma_y})]}{N}$$

standard scores

-15.9 ÷ 11.04

-19.35 ÷ 15.18

(39) From Formula (6), we find that we need the value of $\sum[(\frac{x}{\sigma x})(\frac{y}{\sigma y})]$. This is the ____________ of the product of two standard scores for all subjects. In order to obtain this value a new column headed $(\frac{x}{\sigma x})(\frac{y}{\sigma y})$ is added as follows:

Table 5-b Pearson's r-Standard Score Method

Subject	X	Y	x	y	$\frac{x}{\sigma x}$	$\frac{y}{\sigma y}$	$(\frac{x}{\sigma x})(\frac{y}{\sigma y})$
1	57	142	-15.9	-19.35	-1.44	-1.27	1.83
2	69	148	- 3.9	-13.35	-0.35	-0.88	0.31
3	71	150	- 1.9	-11.35	-0.17	-0.75	0.13
4	60	147	-12.9	-14.35	-1.17	-0.95	1.11
5	80	172	7.1	10.65	0.64	0.70	0.45
6	87	184	14.1	22.65	1.28	1.49	1.91
7	61	150	-11.9	-11.35	-1.08	-0.75	0.81
8	65	152	- 7.9	- 9.35	-0.72	-0.62	0.45
9	46	130	-26.9	-31.15	-2.44	-2.07	5.05
10	77	165	4.1	3.65	0.37	0.24	0.09
11	85	181	12.1	19.65	1.10	1.29	1.42
12	93	190	20.1	28.65	1.82	1.89	3.44
13	71	152	- 1.9	- 9.35	-0.17	-0.62	0.11
14	84	178	11.1	16.65	1.01	1.10	1.11
15	69	155	- 3.9	- 6.35	-0.35	-0.42	0.15
16	79	170	6.1	8.65	0.55	0.57	0.31
17	81	175	8.1	13.65	0.73	0.90	0.66
18	73	160	0.1	- 1.35	0.01	-0.09	0
19	74	162	1.1	0.65	0.10	0.04	0
20	76	164	3.1	2.65	0.28	0.17	0.05
Sum							19.39

$\sum[(\frac{x}{\sigma x})(\frac{y}{\sigma y})]$

(40) Substituting the values obtained from the above table in Formula 6 (Frame 38), we have

$$r = \frac{\sum[(\frac{x}{\sigma x})(\frac{y}{\sigma y})]}{N}$$

$$= \frac{\qquad\qquad}{\qquad}$$

$$= 0.97$$

sum

(41) The Scatter Diagram Method (Grouped Data) When the sample size is large or no desk calculator is available, the observed data may be grouped and the bivariate distribution plotted in a two-way correlation table which is known as scatter diagram. First of all each distribution is grouped into convenient intervals. Usually the number of intervals is somewhere from 10 to 20, and the size of the interval is a convenient figure such as 5 or 10. It is suggested that the lower limit of the lowest interval which is a multiple of 5 or 10 be set equal to or less than the lowest score of the distribution. In our present example, since the lowest schore of the X distribution is 46 (refer back to Frame 37) the lower limit of the lowest interval should be equal to ________. In the case of Y distribution, the smallest score is 130. Therefore, the lower limit of the lowest interval should be set equal to ________________.

$\frac{19.39}{20}$

(42) Let us use 5 as the interval size for both distributions and construct a two-way table like this:

Table 6-a Pearson's r - Scatter Diagram

Y \ X	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89	90 - 94
190-194										
185-189										
180-184										
175-179										
170-174										
165-169										
160-164										
155-159										
150-154										
145-149										
140-144										
135-139										
130-134										

In the diagram, columns represent X - ______ and rows represent Y - ____________. The number of intervals for X is ____________ and the number of intervals for Y is ______ ____________.

45

130

(43) You are now ready to enter tally marks in the scatter diagram. Take one subject at a time from the table of observed data (refer back to Frame 15). For instance, the scores of subject 1 are X = 57 and Y = 142. You would enter a tally mark "/" in column (55-59) and row (140-144) as follows:

Table 6-b Pearson's r- Scatter Diagram

X Y	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94
190-194										
185-189										
180-184										
175-179										
170-174										
165-169										
160-164										
155-159										
150-154										
145-149										
140-144			/							
135-139										
130-134										

Enter the remaining 19 tally marks in the diagram.

intervals

intervals

10

13

(44) After entering all the 30 tallies, count the tallies in each column and record the frequencies in the row headed fx at the bottom of the diagram. Also count the tallies in each row and record the frequencies in the column headed fy to the right of the diagram. Enter your result in the following diagram.

Table 6-c Pearson's r-Scatter Diagram

X Y	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89	90 - 94	fy
190-194										/	
185-189											
180-184									//		
175-179								//			
170-174							/	/			
165-169							/				
160-164						//	/				
155-159					/						
150-154				/	/	//					
145-149				/	/						
140-144			/								
135-139											
130-134	/										
fx											

The result can be found in Table 6-c, Frame 44

(45) Add to the diagram a new row headed d_x and a new column headed d_y. Select an arbitrary origin in Row d_x and another one in Column d_y. Mark "0" in the cell in which the arbitrary origin is located. Then, mark + 1, +2, - - - - and so on to the right of cell 0 in Row d_x and above Cell 0 in Column d_y; mark -1, -2, - - - and so forth to the left of Cell 0 in Row d_x and below Cell 0 in Column d_y. Suppose that the two arbitrary origins have been selected as shown in the following diagram. Complete Row d_x and Column d_y by entering all the necessary deviations.

Table 6-d Pearson's r-Scatter Diagram

X / Y	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	f_y	d_y
190-194										/	1	
185-189											0	
180-184									//		2	
175-179								//			2	
170-174							/	/			2	
165-169							/				1	
160-164						//	/				3	0
155-159					/						1	
150-154				/	/	//					4	
145-149				/	/						2	
140-144			/								1	
135-139											0	
130-134	/										1	
f_x	1	0	1	2	3	4	3	3	2	1	20	
d_x						0						

The result can be found in Table 6-d, Frame 45

(46) Multiply entries in Row f_x by corresponding entries in Row d_x and enter the products in a new row headed f_xd_x. Multiply entries in Column f_y by corresponding entries in Column d_y and record the products in a new column headed f_yd_y.

Table 6-e Pearson's r-Scatter Diagram

X Y	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89	90 - 94	f_y	d_y	f_yd_y
190-194										/	1	+6	
185-189											0	+5	
180-184									//		2	+4	
175-179								//			2	+3	
170-174							/	/			2	+2	
165-169							/				1	+1	
160-164						//	/				3	0	
155-159					/						1	-1	
150-154				/	/	//					4	-2	
145-149				/	/						2	-3	
140-144			/								1	-4	
135-139											0	-5	
130-134	/										1	-6	
f_x	1	0	1	2	3	4	3	3	2	1	20		
d_x	-5	-4	-3	-2	-1	0	+1	+2	+3	+4			
f_xd_x													

The result can be found in Table 6-e, Frame 46.

(47) Multiply entries in Row f_xd_x by corresponding entries in Row d_x, and enter the products in a new row headed $\underline{f_xd_x^2}$. Multiply entries in Column f_yd_y by corresponding entries in Column d_y, and record the products in a new column headed $\underline{f_yd_y^2}$

Table 6-f Pearson's r - Scatter Diagram

X Y	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	f_y	d_y	f_yd_y	$f_yd_y^2$
190-194										/	1	+6	6	
185-189											0	+5	0	
180-184									//		2	+4	8	
175-179								//			2	+3	6	
170-174							/	/			2	+2	4	
165-169							/				1	+1	1	
160-164						//	/				3	0	0	
155-159					/						1	-1	-1	
150-154				/	/	//					4	-2	-8	
145-149				/	/						2	-3	-6	
140-144			/								1	-4	-4	
135-139											0	-5	0	
130-134	/										1	-6	-6	
f_x	1	0	1	2	3	4	3	3	2	1	20		0	
d_x	-5	-4	-3	-2	-1	0	+1	+2	+3	+4				
f_xd_x	-5	0	-3	-4	-3	0	3	6	6	4	4			
$f_xd_x^2$														

The result can be found in Table 6-f, Frame 47.

(48) Add to the diagram a new row headed $(f_yd_y)'$ and a new column headed $(f_xd_x)'$ as shown in the following table. Fill out all the cells in Row $(f_yd_y)'$ and Column $(f_xd_x)'$ by following this example: Take the fourth cell from left in Row $(f_yd_y)'$. Notice that this cell belongs to Column (60-64) within which two tallies are found. One of these tallies belongs to Row (145-149), the other belongs to Row (150-154). Find the deviations of Row (145-149) and Row (150-154) by checking the entries in Column d_y which gives you -3 and -2. Multiply each frequency by its corresponding deviation just found and sum them up. Thus, you have 1x(-3)+1x(-2) = -5. Enter this value in the fourth cell from left in Row $(f_yd_y)'$.

The result
can be fou
in Table
6-g, Frame
48.

Table 6-g Pea son's r-Scatter Diagram

X / Y	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	f_y	d_y	f_yd_y	$f_yd_y^2$	$(f_xd_x)'$
190-194										/	1	+6	6	36	
185-189											0	+5	0	0	
180-184									//		2	+4	8	32	
175-179								//			2	+3	6	18	
170-174							/	/			2	+2	4	8	
165-169							/				1	+1	1	1	
160-164						//	/				3	0	0	0	
155-159					/						1	-1	-1	-1	
150-154				/	/	//					4	-2	-8	16	
145-149				/	/						2	-3	-6	18	
140-144			/								1	-4	-4	16	
135-139											0	-5	0	0	
130-134	/										1	-6	-6	36	
f_x	1	0	1	2	3	4	3	3	2	1	20		0	182	
d_x	-5	-4	-3	-2	-1	0	+1	+2	+3	+4					
f_xd_x	-5	0	-3	-4	-3	0	3	6	6	4	4				
$f_xd_x^2$	25	0	9	8	3	0	3	12	18	16	94				
$(f_yd_y)'$				-5											

(49) Multiply entries in Row $(f_yd_y)'$ by corresponding entries in Row d_x and record the products in a new row headed $\underline{d_x(f_yd_y)'}$ Multiply entries in column $(f_xd_x)'$ by corresponding entries in Column d_y and enter the products in a new column headed $d_y(f_xd_x)'$.

The result can be found in Table 6-h, Frame 49

Table 6-h Pearson's r-Scatter Diagram

X / Y	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89	90 - 94	f_y	dy	f_yd_y	$f_yd_y^2$	$(f_xd_x)'$	$d_y(f_xd_x)'$
190-194										/	1	+6	6	36	4	
185-189											0	+5	0	0	0	
180-184									//		2	+4	8	32	6	
175-179								//			2	+3	6	18	4	
170-174							/	/			2	+2	4	8	3	
165-169							/				1	+1	1	1	1	
160-164						//	/				3	0	0	0	1	
155-159					/						1	-1	-1	1	-1	
150-154				/	/	//					4	-2	-8	16	-3	
145-149				/	/						2	-3	-6	18	-3	
140-144			/								1	-4	-4	16	-3	
135-139											0	-5	0	0	0	
130-134	/										1	-6	-6	36	-5	
f_x	1	0	1	2	3	4	3	3	2	1	20		0	182	4	
d_x	-5	-4	-3	-2	-1	0	+1	+2	+3	+4						
f_xd_x	-5	0	-3	-4	-3	0	3	6	6	4	4					
$f_xd_x^2$	25	0	9	8	3	0	3	12	18	16	94					
$(f_yd_y)'$	-6	0	-4	-5	-6	-4	3	8	8	6	0					
$d_x(f_yd_y)'$																

(50) To compute the Pearson's r, substitute the necessary values given by the scattergram (as shown below) in the following formula:

$$r = \frac{N\sum[d_x(f_y d_y)'] - (\sum f_x d_x)(\sum f_y d_y)}{\sqrt{[N\sum f_x d_x^2 - (\sum f_x d_x)^2][N\sum f_y d_y^2 - (\sum f_y d_y)^2]}}$$

$$r = \underline{\qquad\qquad\qquad\qquad}$$

$$= \frac{2500}{\sqrt{6784960}}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

The result can be found in Table 6-1, Frame 50

Table 6-1 Pearson's r - Scatter Diagram

Y \ X	45 - 49	50 - 54	55 - 59	60 - 64	65 - 69	70 - 74	75 - 79	80 - 84	85 - 89	90 - 94	f_y	d_y	$f_y d_y$	$f_y d_y^2$	$(f_x d_x)'$	$d_y(f_x d_x)'$
190-194										/	1	+6	6	36	4	24
185-189											0	+5	0	0	0	0
180-184									//		2	+4	8	32	6	24
175-179								//			2	+3	6	18	4	12
170-174							/	/			2	+2	4	8	3	6
165-169							/				1	+1	1	1	1	1
160-164						///	/				3	0	0	0	1	0
155-159					/						1	-1	-1	1	-1	1
150-154				/	/	//					4	-2	-8	16	-3	6
145-149				/	/						2	-3	-6	18	-3	9
140-144			/								1	-4	-4	16	-3	12
135-139											0	-5	0	0	0	0
130-134	/										1	-6	-6	36	-5	30
f_x	1	0	1	2	3	4	3	3	2	1	30		0	182	4	125
d_x	-5	-4	-3	-2	-1	0	+1	+2	+3	-4						
$f_x d_x$	-5	0	-3	-4	-3	0	3	6	6	4	4					
$f_x d_x^2$	25	0	9	8	3	0	3	12	18	16	94					
$(f_y d_y)'$	-6	0	-4	-5	-6	-4	3	8	8	6	0					
$d_x(f_y d_y)'$	30	0	12	10	6	0	3	16	24	24	125					

(51) To summarize, the Pearson's r can be computed if the bivariate distribution X and Y are (1) both ________ variables, (2) associated with each other in a ________ fashion, and (3) relatively ________ which means that the two distributions are fairly symmetrical and unimodal. The Pearson's ________ coefficient of correlation is defined as

$$r = \frac{\Sigma\, xy}{N \sigma_x \sigma_y}$$

where (describe the notation)

x = ________________

y = ________________

N = ________________

σ_x = ________________

σ_y = ________________

$$\frac{20 \text{ x } 125 - 4 \text{ x } 0}{\sqrt{(20\text{x}94-4^2)(20\text{x}183-0)}}$$

$$\frac{2500}{2604.8} = 0.96$$

III Biserial Coefficient of Correlation

(52) Now, let us consider the case in which both variables (X and Y) are continuously computable but one of them (say X) is for some reason dichotomized (defined by two categories). If there is reason to believe that the true distribution underlying the dichotomized variable X is of normal form and that the requirements of linear regression and homoscedasticity are met by the bivariate distributions, the biserial coefficient of correlation may be computed. Actually the biserial coefficient of correlation , r_b, is a product-moment correlation coefficient. It is, however, not the Pearson's r, but a good estimate of and a good substitute for the Pearson's r. In order to compute a biserial r, therefore, the same requirements as for the ____________ r must be observed. On top of that the requirement that the dichotomized variable (X) would be ________ if it were distributed must also be met.

continuous

linear

homoscedastic

product-moment

deviation from $\bar{X}$ (mean)

deviation from $\bar{Y}$ (mean)

standard deviation of X distribution

standard deviation of Y distribution

---

(53) For instance, in item-analysis in which the relationship of total test score to a correct-incorrect response to any given test item is studied, the dichotomous characteristic (correct-incorrect) can actually be a normally distributed variable. That is, a normally distributed characteristic is measured by a crude response such as correct-incorrect categories. Of course another variable is the total score which is continuously measured. With other requirements (1. linear regression and 2. homoscedasticity) satisfied, we may conduct an item by item computation of ________________ (with total score) to validate the test items.

Pearson's

normal

(54) Before computing a biserial r, it is suggested that the observed data be grouped in a distribution (table) with 12-20 intervals for the continuous variable along the Y-axis, and with 2 intervals for the dichotomized variable along the X-axis. Suppose that we have a grouped distribution which shows the total test scores and the responses (correct-incorrect) to a particular test item by a group of students as follows:

Table 7-a Observed Data (II)

Y Total Score	X Correct f_p	 Incorrect f_q	Total f_t
90-94	2	0	2
85-89	3	0	3
80-84	5	1	6
75-79	8	3	11
70-74	12	5	17
65-69	10	6	16
60-64	6	6	12
55-59	3	4	[illegible]
50-54	1	3	4
45-49	0	2	2
Sum	50	30	80

There were a total of ________ students who took the test. Of these students ________ answered correctly to this particular test item. There were ____________ students who answered incorrectly to the same item.

biserial coefficient of correlation (r_b)

(55) There are two formulas available for the computation of the biserial r, denoted by r_b. Let us take the first formula:

$$r_b = \frac{(\overline{Y}_p - \overline{Y}_q)\, pq}{s_t y} \text{ - - - - - (III. 1)}$$

where $\overline{Y}_p$ = mean of Y's for cases in the "correct" category

$\overline{Y}_q$ = mean of Y's for cases in the "incorrect" category

p = proportion of cases in the "correct" category

q = proportion of cases in the "incorrect" category

s_t = standard deviation of all the Y scores

y = ordinate of the unit normal curve at the point where p (or q) cases are cut off

Among these six quantities, p and q can be computed directly with the data shown in Table 7-a, Frame 54 as follows:

$p = \frac{\Sigma f_p}{\Sigma f_t}$ = ________________

$q = \frac{\Sigma f_q}{\Sigma f_t}$ = ________________

or q = 1 - p = ________________

80

50

30

(56) The ordinate y can then be obtained from the table of the unit normal curve (see appendix of your textbook). An appropriate portion of the mentioned table is given below:

Table 9 Table of the Unit Normal Curve

Area in larger portion (p)	Area in smaller portion (q)	Ordinate at x/σ (y)	Standard score (x/σ)
0.629	0.371	0.37790	0.3292
0.628	0.372	0.37823	0.3266
0.627	0.373	0.37856	0.3239
0.626	0.374	0.37888	0.3213
0.625	0.375	0.37920	0.3186

From the table, we have

y = ______________________

$\frac{50}{80} = 0.625$

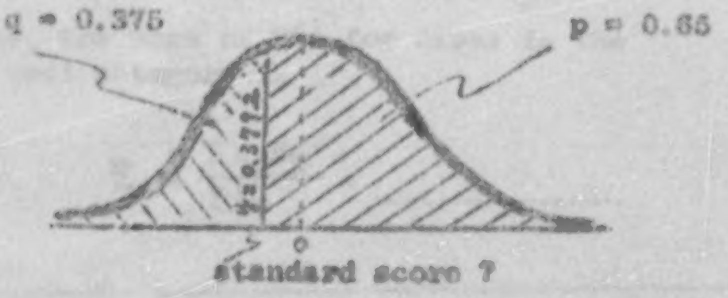
$\frac{30}{80} = 0.375$

1 - 0.625 = 0.375

---

(57) The relation of y = 0.3792 to p = 0.625 and q = 0.375 is illustrated below:

Fig. 14 Unit Normal Curve - y, p, & q



According to Table 8, the point of dichotomy of the x variable has the standard score (x/σ) of ______________

0.3792

(58) We still have to determine $\bar{Y}_p$, $\bar{Y}_q$, and s_t (refer back to equation III. 1, Frame 55). To do this, we must extend Table 7-a (Frame 54). First, let us compute $\bar{Y}_p$. A new column headed Y_i indicating mid points of the 10 intervals are added and these values are multiplied by their corresponding frequencies (f_p). The results are then recorded in a new column headed f_pY_i as shown in the following table:

Table 7-b Biserial r

Total Score Y	Mid Point Y_i	Correct		Incorrect
		f_p	f_pY_i	f_q
90-94	92	2	184	
85-89	87	3	261	
80-84	82	5	410	
75-79	77	8	616	
70-74	72	12	864	
65-69	67	10	670	
60-64	62	6	372	
55-59	57	3	171	
50-54	52	1	52	
45-49	47	0	0	
Sum		50	3600	

Thus, the mean of Y's for cases in the correct category is

$$\bar{Y}_p = \frac{\Sigma f_pY_i}{\Sigma f_p} = \underline{\qquad\qquad}$$

-0.3186

(59) Enter the value $\bar{Y}_p = 72$ in the cell at Row Mean and Column Correct. Add a new column headed f_qY_i to the right of Column f_q and record the products of $Y_i \times f_q$ in that column as shown below.

Table 7-c Biserial r

Total Score Y	Mid Point Y_i	Correct		Incorrect		Total
		f_p	f_pY_i	f_q	f_qY_i	f_t
90-94	92	2	184	0	0	
85-89	87	3	261	0	0	
80-84	82	5	410	1	82	
75-79	77	8	616	3	231	
70-74	72	12	864	5	360	
65-69	67	10	670	6	402	
60-64	62	6	372	6	372	
55-59	57	3	171	4	228	
50-54	52	1	52	3	156	
45-49	47	0	0	2	94	
Sum	50	50	3600	30	1925	
Mean		72				

From this table, we have the mean of Y's for cases in the incorrect category:

$$\bar{Y}_q = \frac{\Sigma f_qY_i}{\Sigma f_q} = \_\_\_\_\_\_\_\_\_\_$$

$$\frac{3600}{50} = 72$$

(60) Record the value $\bar{Y}_q$ = 64.17 in the cell at Row Mean and Column Incorrect. To obtain the standard deviation of all the Y scores, all a new column headed d to the above table. Select an arbitrary origin in an interval and record the deviations from that origin for other intervals as follows:

$$\frac{1925}{30} = 64.17$$

Table 7-d Biserial r

Total Score Y	Mid Point Y_i	Correct		Incorrect		Total	
		f_p	f_pY_i	f_q	f_qY_i	f_t	d
90-94	92	2	184	0	0	2	-4
85-89	87	3	261	0	0	3	-3
80-84	82	5	410	1	82	6	-2
75-79	77	8	616	3	231	11	-1
70-74	72	12	864	5	360	17	0
65-69	67	10	670	6	402	16	-1
60-64	62	6	372	6	372	12	-2
55-59	57	3	171	4	228	7	-3
50-54	52	1	52	3	156	4	-4
45-49	47	0	0	2	94	2	-5
Sum		50	3600	30	1925	80	
Mean		72		64.17			

In this case, we have selected the Interval __________ as the arbitrary origin.

(61) Add two more columns headed $\underline{f_t d}$ and $\underline{f_t d^2}$ to the above table. Multiply entries in Column f_t by the corresponding entries in Column $f_t d$. Then multiply entries in Column $f_t d$ by the corresponding entries in Column d, again, and record the products in Column $f_t d^2$. Thus, the first entry in Column $f_t d$ and $f_t d^2$ shown below are obtained as follows: 8 = ____ x ____; 32 = ______ x______ .

70-74

Table 7-e Biserial r

Total Score Y	Mid Point Y_i	Correct		Incorrect		Total			
		f_p	f_pY_i	f_q	f_qY_i	f_t	d	$f_t d$	$f_t d^2$
90-94	92	2	184	0	0	2	-4	8	32
85-89	87	3	261	0	0	3	-3	9	27
80-84	82	5	410	1	82	6	-2	12	24
75-79	77	8	616	3	231	11	-1	11	11
70-74	72	12	864	5	360	17	0	0	0
65-69	67	10	670	6	402	16	-1	-16	16
60-64	62	6	372	6	372	12	-2	-24	48
55-59	57	3	171	4	228	7	-3	-21	63
50-54	52	1	52	3	156	4	-4	-16	64
45-49	47	0	0	2	94	2	-5	-10	50
Sum		50	3600	30	1925	80		-47	335
Mean		72		64.17					

The standard deviation is then computed as follows (Notice that i denotes the interval size)

$$s_t = i\sqrt{\frac{\Sigma f_t d^2}{\Sigma f_t} - \left(\frac{\Sigma f_t d}{\Sigma f_t}\right)^2}$$

= ____________________

$= 5\sqrt{4.1875 - 0.34515}$

= ____________________

= ____________________

(62) With all these values, you can now compute the biserial coefficient of correlation, r_b, through the use of Equation III. 1:

$$r_b = \frac{(\overline{Y}_p - \overline{Y}_q)\,pq}{s_t y}$$

$$= \frac{\rule{5cm}{0.4pt}}{\rule{5cm}{0.4pt}}$$

$$= \frac{1.335}{3.716}$$

$$= \rule{5cm}{0.4pt}$$

2 x 4 (or 4 x 2)

8 x 4 (or 4 x 8)

$$5\sqrt{\frac{335}{80} - \left(\frac{47}{80}\right)^2}$$

5 x 1.96
9.8

(63) You may use a second computational formula to arrive at the same result. The biserial coefficient of correlation is given, this time, as

$$r_b = \frac{(\overline{Y}_p - \overline{Y}_t)\cdot p}{s_t \cdot y} \text{ - - - - - - - - (III. 2)}$$

where $\overline{Y}_p$ = mean of Y's for cases in the "correct" category

$\overline{Y}_t$ = mean of all the Y scores

p = proportion of cases in the "correct" category

s_t = standard deviation of all the Y scores

y = ordinate of the unit normal curve at the point where p cases are cut off.

Notice that, in comparison to Equation III. 1, we do not have to consider the data concerning the ____________ category.

$$\frac{(72-64.17)\text{x}0.625\text{x}0.375}{9.8\text{x}0.3792}$$

0.49

(64) To facilitate the computation, let us take the necessary columns from Table 7-e, Frame 61 and add a new column headed f_tY_i as shown below:

Table 7-f Biserial r

Total Score Y	Mid Point Y_i	Correct		Total				
		f_p	f_pY_i	f_t	f_tY_i	d	f_td	f_td^2
90-94	92	2	184	2	184	+4	8	32
85-89	87	3	261	3	261	+3	9	27
80-84	82	5	410	6	492	+2	12	24
75-79	77	8	616	11	847	+1	11	11
70-74	72	12	864	17	1224	0	0	0
65-69	67	10	670	16	1072	-1	-16	16
60-64	62	6	372	12	744	-2	-24	48
55-59	57	3	171	7	399	-3	-21	63
50-54	52	1	52	4	208	-4	-16	64
45-49	47	0	0	2	94	-5	-10	50
Sum		50	3600	80	5525		-47	335
Mean		72		69.06				

From this table, values for $\overline{Y}_p$, $\overline{Y}_t$, p, and s_t can be obtained as follows:

$$\overline{Y}_p = \frac{\sum f_pY_i}{\sum f_p} = \underline{\qquad\qquad} = 72$$

$$\overline{Y}_t = \frac{\sum f_tY_i}{\sum f_t} = \underline{\qquad\qquad} = 69.06$$

$$p = \frac{\sum f_p}{\sum f_t} = \underline{\qquad\qquad} = 0.625$$

$$s_t = i\sqrt{\frac{\sum f_td^2}{\sum f_t} - \left(\frac{\sum f_td}{\sum f_t}\right)^2}$$

$$= \underline{\qquad\qquad\qquad\qquad}$$

$$= 9.8$$

The value of y can be obtained from the table of the unit normal curve (see Frame 56) We have, y = 0.3792

incorrect

(65) Substituting these values in Formula III - 2 we have

$$r_b = \frac{(\bar{Y}_p - \bar{Y}_t)\cdot p}{S_t \cdot y}$$

$$= \underline{\qquad\qquad}$$

$$= \frac{1.8375}{3.71616}$$

$$= \underline{\qquad\qquad}$$

$$\frac{3600}{50}$$

$$\frac{5525}{80}$$

$$\frac{50}{80}$$

$$5\sqrt{\frac{335}{80} - \left(\frac{-47}{80}\right)^2}$$

$$\frac{(72 - 69.06)\cdot 0.625}{9.8 \cdot 0.3792}$$

0.49

APPENDIX X

"P.O.L., A GENERAL PURPOSE PROCEDURE ORIENTED COMPUTER PROGRAMMING LANGUAGE"

Abstract:

It is here proposed to develop, implement and document a general purpose, procedure oriented computer programming language, its associated processor and operating system. By "development", the statement of all specifications is meant; by "implementation", the programming and debugging of the processor and the operating system; and by "documentation", the production of manuals, teaching materials, etc., as well as the formalization of the language are understood.

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Most programming languages today are either symbol manipulation or numeric computation oriented. The nature of the computing tasks currently undertaken would seem to indicate however, that such a distinction is artificial, and frequently confining in the selection of problems and the statement of solutions. Attempts have been made to alleviate this condition by placing several languages side by side, under the same processor, each aimed at a different range and domain, a different set of basic algorithms (IBM's IBJOB processor and PL/1 are good examples of this approach). These processors are typified by a user interface that is prohibitive of mastery.

Data structures (*e.g.*, arrays, lists, hierarchical spaces, character strings) have been introduced by the high order programming languages, and today, in every language, at least one type of data structure is definable. Although this is true, a twofold problem exists: few languages present any *variety of data structures* in their domain, and there exists a significant lack of operations *defined over whole structures* (i.e., languages are only defined over the domain of atomic variables, where the atomic may be an element of a structure). There has begun an accumulation of subroutines — defining operations over whole structures -- that is reminiscent of the last days of assemblers before the first high order language.

P.O.L. (for Procedure Oriented Language) has been constructed as a solution for the predicament outlined above. It is a language consisting of a small number of basic operations such that each one of these operations is generalized over both symbolic and numeric as well as both structured and atomic operands. The operand types themselves have also been generalized to allow the representation of a large variety of structures heretofore not easily representable in terms of any language.

In the last few years, programming languages have proliferated beyond manageability by either professionals or academics. In view of this, the development of yet another programming language requires some justification. The cost and effort involved in retraining and recoding for another language comprise only part of the counter-argument. In developing P.O.L., the effort was directed toward creating a language to which these counter-arguments do not apply. P.O.L. is aimed at the professional programmer, not the beginner or the casual user. It is a programmers' language, with very few conventions and mnemonics to remember, but requiring "programming maturity" (i.e., the understanding of some fairly advanced concepts and computing techniques). "Programming maturity" must be thus assumed if a language is to be powe powerful and the user interface is to remain manageably small. The programmers for whom P.O.L. has been developed should find the retraining no task at all.

It is strongly recommended that the casual FORTRAN or COBOL user should not be compelled to retrain and reprogram into P.O.L., because as far as the facilities of P. O. L. duplicate or parallel those of FORTRAN and COBOL, no significant advantage is claimed for P. O. L. Rather, the claims lie in the problem areas and courses of solution that could not be satisfactorily - or at all - approached in other languages. It is dealing with programs of extreme complexity, of large storage requirements, of complex or unusual operation and/or operand types, that P. O. L. is strongly recommended. This consideration should reduce the recoding problem appreciably.

The shape of the language and of the operating system is determined by the modern time and memory shared, multiprogrammed systems, by the "conversational" or "interactive" mode of programming and debugging, by the existence of a variety of storage media of varying speeds and capacities, by the variety of input/output media, etc. Another strong principle that has contributed to the profile of P. O. L. is that of "the complete accessibility of the machine." It has been indicated above that P. O. L. is generalized towards operands and operations of a higher level (e.g., complete structures) than those of other languages. Complete accessibility means a similar generalization toward operands and operations of a lower level, a level that approaches machine language in detail and flexibility. The generalization in both directions is accomodated in the same notation, by the same set of conventions. Complete accessibility also means explicit command over the interrupt (or trap) system and the input/output facilities of the machine. Though the programmer may occasionally prefer to relinquish his responsibility for monitoring the whole hardware in favor of some standardized procedure, this will occur only as a matter of choice, not because the language or the operating system keeps any part of the machine or its capabilities "safe" from the programmers' tampering. The optional, standardized monitoring procedures will be available in the operating system.

THE LANGUAGE

The general appearance of the code is function-form; that is to say that writing a P.O.L. program, one writes a series of functions -- and declarations -- rather than statements, sentences, etc. The value of the P.O.L. function is invariably explicit, each function returning its value in one of its arguments specified for this purpose. In this sense -- that one of the operands refers to the place to store the value -- P.O.L. bears a strong resemblance to machine language.

A program deck consists of two parts: the description of the computational process or algorithm in terms of a rigorous, formal notation system; and a function -- not part of the language but of the operating system -- that causes the execution of the specified procedure. The "functioning structure" of the language is that of an infinitely self-embedding system (i.e., a single function on a given level of embedding may represent a sequence of functions one level deeper), though this is not directly reflected by the syntactic structure of the language.

The description of the computational process consists of a series of one or more macro-function definitions. A macro-function definition in turn consists of a series of one or more functions and declarations, such that any function may either be a primitive or a macro-function. A macro-function definition is delimited by a pair of brackets; the mnemonic DEFINE, followed by a *mention* of the macro being defined, denotes the left bracket; the mnemonic END denotes the right bracket. Macro-functions may not be nested (i.e., a pair of brackets may not enclose another pair of brackets) but any macro-function definition may *use* any macro-function. Legitimate cases of *use* are mutual use (e.g., macro-definition A using macro B, and macro-function definition B using macro A; in other words, defining two or more functions in terms of one another) and recursive use (i.e., defining a function in terms of itself), as well as the general case.

A pair of brackets defines one level of embedding (functional embedding, not syntactic). Objects (e.g., constants and variables) are accessible *only* within the pair of brackets wherein they are declared, *except*:

a) objects defined on shallower levels of embedding that are mapped into the arguments of a given function on a deeper level are modifiable and accessible within that function, just as objects local to that function are.

b) objects defined on shallower levels of embedding that are not mapped into arguments of a given macro on a deeper level, are available within that macro as *terms* in an arithmetic expression or in a predicate or in a source-field, but are not modifiable or alterable within the given macro. Indeed, the only true "guarantee of constant value" in the language, is offered by the use of an object as a term in a *deeper* macro, since the so-called "constants" of the language -- ones declared as such -- are treated as variables with given initial value, and are alterable where available.

A macro-function does not necessarily exit on the function immediately following it; a program may branch from within a macro. Restrictions on branching resemble those on object usage. Both may be subsumed under the heuristic: no inreaching references are allowed, only outreaching ones.

The self-embedding structure described above, as realized by the macro-function feature of P. O. L., is essential for a language that is to operate in a system allowing the "conversational" method of programming. The operating system (to be described below) will permit a program to be "evolved" by defining, executing and debugging one macro at a time, placing the macro in temporary storage on the system, and later defining, executing and debugging macros that use the ones in temporary storage, until the final form and extent of the program is reached.

The operating system that is to host P. O. L. is constructed around a dynamic storage allocator (DSA). In principle, all objects of a given program are allocated dynamically when the macro in which they are declared is entered, and are deleted (and the space they took is returned to the DSA as space available) when exit from that macro is taken. In reality however, only those objects that take more space than does their description, and those that may vary in size and shape from one execution of the macro to the next, will be dynamically allocated; others will be allocated at compile time. The use of this feature will provide for much more efficient use of the storage than would be possible without the DSA.

In addition to its function in handling the objects of the language, the DSA will also handle such tasks as the loading of programs, the sharing of storage between several programs, relocation of programs, and debugging. These activities of the DSA will be discussed in a subsequent section, devoted to the operating system.

The admissible operands of the language are either atomic or structured. Atomic operands may range in size from one binary digit to storage size (subsuming possibly several storage media), and are characterized by having no formal substructure. Portions of atomic operands may be accessed by byte definitions. Any continuous subfield of an atomic operand (regardless of word length or other specifics of the machine) may be defined as a byte; however, bytes must be contained in their entirety by an atomic. Several bytes defined within a given atomic may overlap (i.e., include a particular portion of the atomic). Atomics that are declared as atomics, atomic elements of structures (i.e., subscripted variables) and bytes count as admissible atomic operands. Bytes may be declared as portions of subscripted variables as well as of "stand-alone" atomics. The former case yields the notion of the subscripted

the designated byte.

The above are the machine-independent admissible atomics. A variety of machine dependent or machine specific atomics may be made available in various implementations of P.O.L. In the implementation discussed here (for the CDC 3600) several registers and external switches will be made available. These include registers of the interrupt system, the hardware clock, and the jump, stop and sense switches. Some of these, depending on their nature as determined by the hardware, can only be

accessed, but not altered. Other special atomic operands with limited occurrence will be described in the context of their occurrence.

The structured operands admissible in P.O.L. fall into four classes of explicitly definable objects: arrays, character strings, descending (or common) lists and symmetric lists. These are essentially representation classes; the array structure may be used to represent vectors, vector spaces, matrices, systems of coordinates, tensors, etc., and the various list and string types may represent vectors, graphs, trees, strings, rings, lattices and a multitude of other structures. Emphasizing the nature of these structures as representation classes is in accord with the procedure orientation of P.O.L.

Some of the more interesting or unusual features associated with the admissible structured operands of P.O.L. shall be highlighted below.

Arrays may be defined such that the characteristic parameters such as n (the number of dimensions or the dimensionality of the array), $L_1, \ldots L_n$ (the length, meaning the number of elements along each dimension, specified independently), $O_1, \ldots, O_n$ (the index origin for each of the dimensions, specified independently), and s (the element-size in the array, specifiable regardless of wordsize on the specific machine), are all specifiable by terms in the macro; meaning that each characteristic parameter of an array, as declared in a given macro-function definition, is computable within functions on lower levels of embedding. That in turn means that each time a given macro is executed, a given structure local to it may assume different size and shape. Once the given macro is entered

however, no alteration of the parameters is possible, except as described later in the discussion of the primitive functions SET and MOVE.

Similar to the notion of a byte, as part of an atomic operand, is the notion of partial operands or substructures as parts of structures. Any contiguous part of an array, having the same or lower dimensionality, the same or shorter length along corresponding dimensions, corresponding or non-corresponding index-origin along corresponding dimensions, but always identical element-size, is declarable as a sub-array. Sub-arrays may be operated upon as explicit operands, without altering the elements of the containing array that are not also elements of the sub-array.

Another powerful facility of P.O.L. is "nested indexing", allowing the subscripting of subscripts (i.e., any subscript itself may be expressed as a subscripted variable) to any depth.

When subscripting, the index-origin may be "suppressed" for any given operation, by forcing one-origin indexing without altering the specified index-origin.

A particular type of list, the character string is an explicit operand-type in P.O.L. Strings are symmetrically linked, one-alphanumeric-character-payload lists that are manipulable by all the operations of the language that apply to lists, but in comparisons and tests (i.e., predicates) they behave as atomics, not as strings of discrete elements. This feature has been included in the language with particular regard for the requirements in language data processing applications.

Other lists may either be symmetrically or descendingly linked. Descending lists are single linked, the link pointing away from the head-cell, symmetric lists are double linked, one link pointing toward, the other away from the head-cell. The element size must be constant throughout a descending list, and may be specified as either constant or variable in a symmetric list. Variable element-size means that each element of such a list may be of a different size, to be specified when the specific element is attached to the list. The specification of characteristic parameters for lists and strings is according to the same principles that were discussed as related to arrays. List elements are accessed by subscripting. A subscripted variable may either be an element of an array, or an element of a list or string. Nested indexing applies to lists as well as to strings, and in a given nest, some values may come from lists, others from arrays, others yet from atomics or bytes.

List elements may be added or removed anywhere on the list, not only at the top and the bottom of the list. This facility subsumes the conventional first-in-first-out and first-in-last-out lists in addition to the less restricted cases.

In their machine representation, lists are threaded; that is to say that for all lists and strings, the head-cell points at the next cell and also at the currently last cell of the list, and the last or bottom cell of the list points back at the head-cell. This does not make the lists circular access (i.e., the index-value one greater than the one corresponding to the bottom, corresponds to the top of the list), all lists in P.O.L. are linear access. Threading is included merely to speed up list processing.

A number of novel and powerful features have been incorporated in P.O.L. in relation to the branching of lists, making available to the programmer a flexible facility with which to represent extremely complex structures, and to operate on them with reasonable ease. Branching in P.O.L. may be n-ary, not only binary; that is to say that any number of branches may be attached to the same branch-point. Any list type may be branched off any other type (types meaning symmetric, descending lists and strings).

Two kinds of branching in lists are specified in P.O.L.: coordinate and subordinate. Coordinate branches may be regarded as lattices, i.e., partial orderings over sets. To illustrate: let there be a coordinate branch structure such that A is the "stem" or "trunk" and B and C are its coordinate branches, yielding an upsidedown Y shape. Let S be the set comprised of the subsets A, B, and C. It can now be said, that the subsets A, B, and C are all internally ordered; that both B and C are ordered with respect to A but not ordered with respect to one another. These ordering considerations explain the "traffic rules" associated with coordinate branching, e.g., a coordinate branch may not be entered by successive increases of the index value as moving down the trunk (beyond the last element on the stem, elements are undefined) but the stem may be entered by successive decreases of the index value, as traveling up a coordinate branch. This traffic restriction applies even if there is only one branch attached to the end of the trunk and no ambiguity exists about the identity of the (n+1)th element of an n long list. Subordinate branching may be described by either of two formal devices: a sublist is either a partial or complete extension of the set designated by the cell to which it is attached, or it is a partial or complete list of the properties of the cell to which it is attached. Either of the two formalizations places the trunk and the sublist on separate levels, explaining the fact that main and sublist are mutually inaccessible through indexing along one or the other. Any list type may be either kind of branch of any list type. The decision to use symmetric or descending lists should be made on the bases of storage requirements and desired speed: operations are performed faster on symmetric lists, but the double linking takes up considerably more storage. Both types of branches may be intermixed from the same branch-point.

The specification of objects in P.O.L. is done in terms of declarations. These are non-sequential elements of the program (meaning that they can occur anywhere within the macro-function definition in which they apply) that correspond to the "Let P be ..." sentence form of mathematical and logical notations. Their form in P.O.L. is:

LET, name, designator, specification, /contents/*

where "LET" is a mnemonic, identifying this statement form; "name" stands for the proper name of the object being defined (it is by this name that the object, or parts of it are referenced throughout the program); "designator" identifies the operand type (ATOM, ARRAY, STRING, LISTD and LISTS are the mnemonics that may appear in this position); "specification" stands for the parameters describing the object, as discussed above; "contents" identifies the object as a constant, which in P.O.L. means a variable with its initial value specified (i.e., the compiler does not prevent altering the value of a constant). It is in the "contents" portion that the only reference is made to representation (e.g., hollerith, octal, fixed or floating point, etc.). Representation is not part of the description of the objects of the language. This will be further discussed when describing the SET primitive. Structures as well as atomics may be defined as constants. Literals are specified in the form

designator, specification, contents

In addition, there are a number of literals that are likely to be of frequent use, and have been made part of the language, specifiable in terms of simple mnemonics. These are: full vector, e.g., (1,1,1,1), interval vector, e.g., (2, 3, 4, 5, 6), j-th unit vector, e.g., (0, 0, 0, 1, 0), prefix vector, e.g., (1, 1, 0, 0, 0), and suffix vector, e.g., (0, 0, 1, 1, 1), left unit matrix, e.g.,

$\begin{matrix}1,0,0\\0,1,0\\0,0,1\end{matrix}$, and right unit matrix, e.g., $\begin{matrix}0,0,1\\0,1,0\\1,0,0\end{matrix}$. These operands are only usable as one of the two operands of a binary operation, and the size, shape and designation as well as the representation of these special literals will conform automatically to that of the other operand.

P.O.L. has nine primitive functions: SET, MOVE, IF, COND, GOTO, PAUSE, READ, WRITE and POSITN. It is in terms of these primitives that any macro-function, and on a higher level, any algorithm coded in P.O.L. is computable. In creating and specifying these primitives, the primary guiding principle was generality, that is to say making available as many capabilities as possible in terms of as few conventions and mnemonics as possible. The two other major guide lines used for the development of the language were intuitiveness and formalizability. By the first it is meant that if several capabilities are placed under the same primitive function, there should be some intuitive relationship between them, so that upon cursory reading of a manual or other document on P.O.L., a programmer would be able to match capabilities with primitives on intuitive grounds, not on the bases of memory; the latter means, that all other things being equal, solutions and courses of action were adopted that formalize easier.

On some level of sophistication, most programming languages have the two statement forms: "set the value of A equal to B" and "move the contents of B into A". These are often translated into the same object code, and invariably produce the same results. In P.O.L., this overlap of manipulating and computing facilities has been eliminated. The separation is not along the lines of "arithmetic" vs. "manipulating" but in terms of "value operations" vs. "entity" operations". This serves as both intuitive and formal grounds for the distinction between the role of the primitives SET and MOVE.

SET is the primitive function of the language that performs the value operations. Its shape in the notation is:

/label/, SET, (1st argument$_1$), . . . , (1st argument$_n$), (2nd argument)

where the label is optional and provides the reference for possible branching to a given use of the function; SET is the mnemonic identifying this primitive, the first argument is some object of the language and the second argument is some arithmetic expression. The function sets the value of the first argument to equal that of the second argument. There may be any number of first arguments.

A large number of operators are available for the creation of expressions in the second argument. Unary operators are, among others, absolute value, trigonometric functions, logarithms (both natural and ten base), conversion operators (altering the representation of the object e.g., from fixed to floating point), logical NOT. Among the binary operators are the standard arithmetic operators: plus, minus, multiply, divide, exponentiate (for all rational exponents). As will be explained later in more detail, the set of arithmetic operations will be available in several modes (e.g., integer, fraction, floating point single and multi precision, etc.). Binary logical operations in SET will include AND, OR, exclusive OR, implication and equivalence, the relational set of operators consists of greater than, greater than or equal, equal, not equal, less than, less than or equal (the domain of the relational operators is as inclusive as the domain of the rest, but their range is 0 and 1, and structures in 0 and 1).

The syntax of the expression in the second argument is relatively simple: all operators may be freely intermixed to form an expression; logical and relational operators appear in the full parenthesis notation (i.e., every logical and relational operator is bracketed with its operands) and arithmetic operators occur in the standard, elided parenthesis, arithmetic notation.

Two unusual unary operators in SET are the rotation and the count operators. Rotation may be end-off or circular, and either may apply to both structures and atomics. In the former case, any well formed vector or diagonal of an array (i.e., any set of spatially contiguous elements in a spatial model or realization of the array) or any linear body of list (or part of list) may be rotated; in the latter, the operators apply as shift operators and may operate on any atomic or byte of atomic, regardless of size as compared with the wordsize of the machine. The count operator also has several modes of operation. In essence, it is used to count all countable attributes of the objects in P. O. L. (e. g., length of a list or a vector, the number of branches of a given type originating from a branch-point, etc).

It has been stated above that representation is not one of the essential properties of an object in P. O. L. That means that the mode of the operator is entirely independent of the mode of the operand (e. g., fixed point arithmetic may be performed on a floating point object, etc.). All arithmetic is defined over vectors of binary digits, not on values in specific representations, creating a facility as flexible as machine language for performing manipulations.

Two levels of operations are defined: between structures and between atomics (operations obtaining between an atomic and a structure are legitimate, but are interpreted as extensions of the atomic into a structure that matches the other operand). Operations between atomics have been discussed above. Operations between structures may be explained either by extending the operator into a structure, thus creating a structure of expressions, or by the iteration of a given operation through the operand structures' elements. Operations are not restricted to apply between structures of analogous shape, size or even type (e. g., a list and an array may be multiplied). Traffic rules through the structures are in terms of increasing index values, and are specified sufficiently rigorously that shape, size and type restrictions need not apply. Two modes of structure-handling are available: the normalized and the truncated modes. In the normalized mode (which is assumed by the compiler if no specification of mode is given) the smaller operand, or if need be both operands, are extended by adding additional elements that contain the identity element for the specific operation, until the two operands have the same number of elements. In the truncated mode, the larger structure is truncated to match the element-count of the smaller one. Operations between structures may be justified on any pair of elements (when nothing is specified, justification is assumed on high-order elements). In case of unusual justifications, the normalization or truncation works both at the high and the low order end of the structures.

The only restrictions on the first argument of SET are: for an expression in atomics, the first argument must be atomic; for an expression in structures, it must be a structure; the element-size and element count of the structure or the size of the atomic in the first argument must be large enough to accomodate the value of the expression in the second argument. In the case of multiple first arguments, the value of each will be set equal to that of the expression.

Another form of the SET primitive will be discussed below, where the conditional expressions of the language are described.

MOVE is the primitive function of the language devoted to the "entity" operations. Its shape is:

/label/, MOVE, (1st argument), (2nd argument$_1$), . . . ,(2nd argument$_n$)

where the first argument usually denotes the "source" and the second argument the "target". Where such a notion is meaningful, there may be any number of targets; in some of the other cases the first or the second argument may function as a "qualifier" of the function.

The intuitive cohesion of the capabilities included in MOVE may be less readily apparent than it was for the primitive SET: in fact MOVE may be the most difficult primitive in P.O.L. The rationale for this primitive is that it provides a means of altering some of the attributes (but not values) of objects that have been established by declarations. The facility corresponding to "move the contents of A to B" is provided by the primitive SET. The various capabilities in MOVE will be presented one-by-one, and their relation to the central concept will be indicated.

Sets of one or more elements are added to lists or strings by the schema:

$$\text{MOVE (A) (B, UNDER, } \genfrac{}{}{0pt}{}{/\text{LIN}/}{/\text{STR}/}\text{)}$$

where A indicates the source-field and B the target. The following are well-formed cases of this MOVE-type:

if A is an atomic (of any sort), B may be the name of a list, in which case the new element will be added to the bottom of the list below what was heretofore the last element; B may be the name of a list followed by the mnemonic ORG, which causes the new element to be added at the top of the list, "pushing down" the rest of the elements; or B may be a subscripted variable from the list (i.e., the name of one specific element on the list), that results in the new element being inserted on the list below the named element. The "LIN" or "STR" options are not applicable in this case. If A is an atomic from another list that is a branchpoint on that list, and if when attaching A to B, all the branches are also desired, the "STR" option will indicate the desire to bring the structure along with the element.

If A is a list-name or the name or designation of a partial list, B may be any of the three things indicated before. In each case, essentially the same thing will happen as when A is atomic, except several elements will be added to the list rather than just one. The "LIN" option here means that only the list body is to be added onto B, the "STR" option indicates that whatever branching may be associated with A is also to be duplicated for the new section of B. Either LIN or STR must be stated. This form of the function always results in duplicating the source in the target area. Any number of targets may be designated.

Branches are attached to a list by the schema:

$$\text{MOVE (A) (B} \begin{matrix}/\text{SUB}/\\ /\text{CO}\ /\end{matrix})$$

where A indicates the "source", namely the list that is to become the branch, and B indicates the list and the location where the branch is to be attached. The mnemonics SUB and CO indicate the type of branch that is to be created. This form of the function invariably results in "attachment" rather than "copying"; ~~thus the LIN~~ and STR options are not available. Depending on the type of branching desired, the following forms of the function may be used:

if the CO option is taken, indicating coordinate branching, A will be a list-name, the name of the list that is to become a branch; B will also be a list-name (unqualified), because coordinate branches attach to the last element of the list by definition.

if the SUB option is taken, indicating coordinate branching, A will be a list-name, that which is to become the branch; B however will be the name of an element in a list (subscripted variable), the name of the desired branchpoint, as any cell can be a branch-point for subordinate lists. A "source" may be simultaneously attached to any number of lists as a branch, by specifying multiple "targets".

The branches that are to be attached must be separately declared lists in P.O.L. This restriction makes it possible to operate on every branch, either as a branch or as an independent list, without affecting the "trunk" or the state of attachment of the branch.

In the process of constructing P.O.L., the problems of writing very large programs, where one portion of the program may not be "cognizant" of the activities of other portions, have been kept in mind, and accommodated wherever possible. The "count" operator in SET was one such provision; the facility described next is another.

If there is no information about the extent and nature of a list-structure, that structure may be analysed and its branches (and the branches of the branches) may be made accessible by the following schema:

MOVE (ACCESS) (B)

This schema is bound as to its occurrence to the left-most position in the second argument of an IF, and there only following a given predicate-form. That restriction is to be further described with other features of that primitive. The "source" of this move is implicit in this schema and is stated in the IF wherein the MOVE occurs. The concept behind the ACCESS MOVE is the identification of branches from a given branch-point for future accessing. B is the name of an empty list. The execution of this form of the move will place upon list B, as its elements, the "access addresses" of the branches attached to the given branch-point. An "access address" in P.O.L. acts essentially as a message-carrier: through it the branch may be "indirectly addressed" without knowing the proper name of the list that became that branch. To illustrate, let A be the name of the branch whose name is unknown to this portion of the program, but whose access address has been determined and placed on list B by this operation. Elements of A may then be accessed as operands in the general domain and range of P.O.L. by the form "$B (subscript$_1$) (subscript$_2$)" where "$" indicates indirect addressing, "subscript$_1$" identifies the element of list B where the access address is, and "subscript$_2$" identifies the element of list A that we wish to operate upon. Leaving off the second subscript would indicate that the operation is to be performed on list A in its entirety. Adding a third subscript would indicate that the operation is to be performed on the stretch of A that is bounded by the subscript values of the second and the third subscripts. Only one item may be accessed at a time and only one access address may be created for it.

The schema: MOVE (*A*) (DETACH)

serves to detach branches from a list, not altering the value of either trunk or branch, merely moving the attachment of one list to the other. *A* may either be the name of a list or the name of one element of a list. Branches attached to the named object will be detached: in the former case, all branches from the whole body of the named list, in the latter, all branches attached to the named cell will be detached.

Detaching a specific branch is done in terms of the schema:

MOVE (*A*) (B, DETACH)

where *A* is the name of an element of the list (the branch-point) and *B* is either the proper name or the access address of the specific branch that is to be detached. In the DETACH schemata, only one "target" may be specified.

As it has been indicated when discussing declarations, the shape and the size of declared elements are frozen for a single execution of the macro. The MOVE schema described below is the only means of altering size and shape:

$$\text{MOVE } (\underline{A}) \text{ (ERASE } \begin{matrix}\text{/LIN/}\\ \text{/STR/}\end{matrix}\text{)}$$

where *A* may stand for part or all of any structured operand. Any contiguous part of a list may be designated by *A*; if such is the case, one of the two options (STR or LIN) must be selected to indicate whether the attached branches are also to be erased or are only to be detached when the branch-point is erased. In either case, if there exists an access address for any of the branches which are detached and/or erased, that access address will also be erased.

A may also designate a portion or all of an array. In the former case the erasable portion is restricted to spatially contiguous sections (spatially contiguous in some spatial model or realization) of the array, such that along each dimension, the "layer" to be erased is to contain the highest index-value element of that dimension. To illustrate, let A be an array whose shape is (10 * 10 * 10 * 10), i.e., a four dimensional array, ten deep in each direction. If the primitive of the form:

MOVE (A (3 * 2 * 0 * 5)) (ERASE)

is applied to the array, the resulting array A will be of the shape (7 * 8 * 10 * 5), containing the originally high order elements.

References made to erased elements will be undefined, and will result in an error condition. For further discussion of such error conditions, see below under the exposition of the function COND and that of the operating system. Upon erasing elements from lists, new ones may be added at any time while the list is in existence, but no new elements may be added to an array. Arrays may only be altered by removing elements, not by adding new ones. A structure may be completely emptied either by repeated partial erasures or by one total erasure (which does not make references to the whole structure invalid but only those references to its elements.)

The last schema in MOVE is not likely to be used very often, and when used, will require extreme care. It is of the form:

MOVE (A) (B, LINK)

and is designed to create "convergence" in lists. Convergence means that one list merely continues on the body of another, without a branchpoint. In the case of descending lists, the last cell of the list that is to converge into another is provided with a link pointing to the target cell in the other list (the other list is not altered at all). If the converging lists are symmetric lists, the down-pointing links converge but the up-pointing link of the target-list is un-altered;

5

upward the list does not diverge. In the schema above, A is the name of a list-element, the last element of the list that is to converge into the other; B is the name of a list element in the target-list, the point where the other list converges into it. A list may converge into another element upon itself, or into another list.

Convergence represents a borderline case in P.O.L., in that it was considered worthwhile to incorporate it in the representation to allow additional flexibility, but no arithmetic or other manipulation has been defined for convergence, and there is no means of testing for it. It is believed that the need for convergence will arise seldom, and under unusual circumstances, so that users of this feature will want to specify their own functions and their own housekeeping to handle convergence.

IF is the language's function available to specify conditional forms. The IF primitive is part of the program sequence, and corresponds to the conventional "if ... then ..., else ..." statement forms of other programming languages. the IF function is defined over any number of pairs of arguments. The schema is:

/label/, IF, (1st argument$_1$) (2nd argument$_1$)
(1st argument$_2$) (2nd argument$_2$)
. .
. .
. .
(1st argument$_n$) (2nd argument$_n$)

where the first arguments are predicates and the second arguments consist of sequences of functions that specify some course of action. The predicates consist of elementary predicate forms which may be combined into complex predicates by the use of logical connectives. The execution of an IF function results in the sequential examination of the first arguments until the first one is located that is true; then, the corresponding second argument is executed. The second argument may consist of or terminate in a branch function, which causes some alteration of the program sequence; if neither is the case, the IF function will exit (regardless of the order or place of the argument executed) outside the arguments of the IF on the function immediately following the IF.

The following four schemata generate well-formed elementary predicates:

1) (t R t) where the "t"'s denote objects of the language and R is a relational operator. The objects may be atomics or structures. If either or both of the objects are structured (other than strings), the relational operators may be applied in two different ways: "all are" or "some are" (e.g., all elements of A are greater than B; some elements of A are greater than B).

2) (list-name, $\begin{matrix}/CO\ /\\ /SUB/\end{matrix}$, element-name) whose meaning is "the list designated by the name is a sub/co-ordinate branch from the branch-point designated by the element-name".

3) (element-name, $\begin{matrix}/SUB\ /\\ /CO\ \ /\end{matrix}$) means "there are some sub/co-ordinate branches off the element designated by the element-name". It is this predicate that occasions the use of the access form of MOVE (i.e., only if the first argument contains an elementary predicate of this form may the access-move be used). The element named in the predicate designates the branchpoint that is to be accessed.

4) In addition to the above forms, the predicate may consist of a function whose domain is all objects of the language but whose range is 0 and 1 (a Boolean function). Boolean functions may not be combined

with other elementary predicate forms to form more complex predicates. The value one will be taken to correspond to true, and cause execution of the corresponding second argument.

The second argument may consist of any meaningful sequence of one or more functions (primitive or macro). That means that there may be an IF in the second argument of an IF (nesting of IFs). IF's may be nested to any depth. In nesting, it is very important to take in account the traffic-rules: unless otherwise specified, every IF exits on the next statement following it on the same level of nesting.

COND is a conditional quasi-function which establishes condition-procedure pairs such that whenever subsequently the event referred to in the condition occurs, the procedure is executed. The COND may be used to control the hardware interrupt (or trap), to control the internal and external indicators of the machine, to specify the course of action if the dynamic storage allocator runs out of space to allocate, and to establish a programmed software interrupt system. The COND quasi-function is non-sequential (i.e., it is not part of the program sequence; just as in the case of declarations, the effects of the COND are for the duration of the macro, wherever the COND occurs).

The COND quasi=function is defined over any number of pairs of arguments, and takes the form:

$$\begin{array}{lll} \text{COND,} & (\text{1st argument}_1) & (\text{2nd argument}_1) \\ & (\text{1st argument}_2) & (\text{2nd argument}_2) \\ & \vdots & \vdots \\ & (\text{1st argument}_n) & (\text{2nd argument}_n) \end{array}$$

where the second argument may be any meaningful sequence of one or more functions (primitives, including IF's, as well as macros). The traffic is different from that in the IF's, due to the non-sequential nature of COND. At the end of the procedure specified in the second argument which is executed, unless otherwise specified by a branch, the program will resume where the interrupt occurred.

The first argument may assume the following shapes:

(ALLOC, data-name, condition-label)

where ALLOC is a mnemonic that identifies this conditional to refer to space shortages associated with the dynamic storage allocator; the data-name designates the object that this space shortage references, and the condition label identifies this particular interrupt event for reference elsewhere (see below).

(INRPT, interrupt-mnemonic, condition-label)

where INRPT is the mnemonic that identifies this conditional to refer to the state of the hardware-interrupt system; the interrupt-mnemonic identifies the particular interrupt-condition that is referenced (e.g., signal on channel, bounds error, time-limit, etc). This form of COND allows the programmer to write his own trap routines and to have different routines associated with a given interrupt condition for every macro. The condition-label functions as above.

(UNDEF, data-name, condition=label)

where UNDEF is the mnemonic that identifies this conditional to refer to references to undefined objects (e.g., subscript values that do not reference elements); data-name identifies the *object* to which the undefined *element* reference is made; condition-label, as above.

(BOOL, predicate, condition label)

where BOOL is the mnemonic that identifies the condition-type; the predicate may be any predicate that is admissible as the first argument of an IF argument-pair; condition-label, as above. This facility allows the specification of hardware-independent interrupt-conditions; a predicate is "declared" which causes an interrupt whenever in the course of the macro it reaches the state "true". In iterative and recursive procedures (i.e., where the value of an object may pass beyond desired limits), such a software interrupt facility is extremely useful.

Since the COND is a non-sequential quasi-function, there is a need for some sequential control over the conditions declared by it. The form of the SET:

```
                                      /ENABLE/
/label/SET (condition-label) (   /DISABLE/  )
                                      /DELETE /
```

manipulates the COND control of the program and the hardware. DISABLE "turns off" the condition designated by the label in the first argument, ENABLE "turns on" the condition, and DELETE erases the condition specification altogether. The occurrence of a "trap" or "interrupt" will disable the condition-clause associated with it, and it must be ENABLEd again if another interrupt is desired upon recurrence of the event. On some occasions, it is desirable to suspend for some period the effect of a given clause; in that case, it must be DISABLEd. If a condition occurs which is hardware-interrupted or system-interrupted (e.g., DSA problems, undefined references, etc.) and if either no condition clause has been specified for it or the corresponding clause is disabled or deleted, the standardized procedures incorporated in the operating system will go into effect. Those procedures are discussed further under the operating system.

The two control primitives of P.O.L. are GOTO and PAUSE. Both are unconditional, since any conditional forms of them may be constructed in conjunction with the IF and the COND functions. Both compile as single machine instructions; both of them may be interrupted if so specified in the COND. The GOTO function is of the shape

/label/ GOTO, (label)

where the argument refers to the label of the function where the branching is indicated; PAUSE has no arguments and is of the form:

/label/ PAUSE

There is one special case of the GOTO: when the branch-address is a data-name rather than a function-label. Such a reference to data-names is legitimate in P.O.L. to permit the execution of programs written in other languages, and to read them in as data. A GOTO reference to a data-name will cause the GOTO to compile as some form of a return-branch (i.e., the branch will receive, through an index-register or some other means, the re-entry address).

The input/output facilities of languages and the associated operating systems have long been the most controversial area of language and system development. It is anticipated that there will be many modifications of P.O.L. prepared and used by various installations, adapting the i/o facilities to the house policies. The original set of i/o functions will however reflect the principle expressed earlier, that of the complete accessibility of the machine. The three primitives READ, WRITE and POSITN provide the input/output and non-transmit positioning in all i/o devices and storage media (e.g., drums and disks, tape, inter-machine interface(s), plotters, printers and keyboard devices, CRTs and lightpens, etc.). Their shape is

/label/, READ, (device name, count) (data-name)
/label/, WRITE (data-name, count) (device name)
/label/, POSITN (device name, count);

these primitives will be further discussed under the portion devoted to the operating system.

A variety of "benchmarks" has been introduced for the evaluation of programming languages, such as their ability to serve as the meta-language of the compiler or of the operating system. These seldom yield very good measures of the qualities of the language since there are great differences between the sophistication and complexity of the various processors and operating systems. P.O.L.'s compiler and operating system will be written in P.O.L., but this fact is not regarded as a significant benchmark.

To digress for a few lines toward a general point, it is strongly believed by the present investigators that problem oriented languages do not represent the correct path for present and future language development. There are new fields of computer applications being discovered constantly, and the specialists

of the new fields find it cumbersome to use a problem oriented language developed for and couched in the terminology of another problem area. Also it has been found that methods developed in one discipline are often useful in another; problem oriented languages make such interchanges difficult. Additionally, with the proliferation of problem areas amenable to computer solutions, a corresponding proliferation of programming languages may be expected if that path is followed. The answer seems to lie in the direction of a general purpose, procedure oriented language (such as P.O.L. is) with the ability to express problem oriented languages in the terminology of the procedure oriented language.

The macro-function structure of P.O.L. is eminently suitable for the development of problem oriented sets of macros that may be included in the processor. This would result in the evolution of any number of problem-oriented languages but all of them would be couched in the notation of a general, procedure oriented language. More will be said about this under the discussion of the compiler and the operating system. It is sufficient to say here that one benchmark adopted for the evaluation of P.O.L. is the ability of the language to express problem oriented sets of functions.

THE COMPILER

The compiler of necessity reflects the considerations that conditioned the development of the language. Some of these considerations are:

the general usability of the language, as reflected partly by merits of the language itself, and partly by such characteristics as speed of compilation, the efficiency of the object code and the storage utilization, the flexibility of the

compiler and its translatability (i.e., the ease of implementing the system on a new computer);

the suitability of the language and the processor for the conversational or interactive mode of programming;

the ability of the language to accommodate the problem-oriented subsystems developed in terms of it.

Syntax-directed compilers were originally proposed as a step toward language-independent, universal compilation. A by-product of the work on syntax-directed compiler algorithms was the discovery that such compilers can be both faster and smaller than their predecessors, the statement-by-statement processors, due to their orientation toward the general rather than the specific case. A similar by-product of the automatic processing of natural languages was the realization that schemes in which the syntax is implicit in the program (rather than explicit in some grammar-table) tend to be much faster in their machine realizations. An additional advantage of the syntax-directed methods is the ease of constructing diagnostics of well-formedness. Consequently, the compiler for P.O.L. will be a syntax-directed processor with the grammar implicit in the program. Furthermore, it will not function in terms of an intermediate language but will produce direct translations from source language to binary machine code.

Though the distinction between interpreters and compilers has been disappearing, the contemplated P.O.L. processor is considerably closer to the classic compiler notion than to that of the interpreter. This distinction is made from the point of view of the system-design, not from the point of view of the user.

The object-code corresponding to the primative functions has been analyzed and small "building blocks" have been constructed. The code corresponding to a specific function will be built up from these blocks. The blocks consist of sequences of machine instructions which are common to several functions and which will be inserted whenever proper.

The compiler, in its basic form, will recognize primitive functions and generate the macro-functions that are defined in the program. In addition, there will be a temporary status associated with macro-processing, such that the compiler will retain the structure of a given macro for future use, but this "definition" will not become an integral part of the compiler. Macro-definitions that are used frequently over a period of time can be given system macro status, such that the macro-function definition becomes an integral part of the compiler, and that macro will be available for use by any user of the system. A macro in temporary status is only available to the user who defined it and placed it there.

The compiler will be geared primarily toward the generation of programs that will be kept, in some form, on the system library, and will be called into execution by some code issued from a remote console. Programs may also be generated on punched cards, paper-tape or magnetic tape, either in a form that can only be executed by the P.O.L. system, or in a self-contained, self-loading version. The self-loading program will, however, assume complete command of the hardware and will not function properly under another monitor.

In the past, the development of a compiler and an operating system meant immense expenditure of both funds and time. More recently the situation has changed considerqbly. Pioneered by the first self-compilers (such as NELIAC), followed later by such efforts as the META II and META III compilers developed at UCLA and Chapin's IPL-V compiler, the development of compilers is a much cheaper and shorter-term effort today. The P.O.L. compiler will be developed in a series of approximations. The compiler will be written in P.O.L. By using some compiler like one of the META systems (for documentation on META II and III as well as on Chapin's IPL-V processor, see the 1964 ACM Conference Proceedings), another system will be generated; then, in terms of that, another, until a compiler will be generated that is powerful enough to generate the P.O.L. compiler. Then, P.O.L. will be recompiled by the self-compiler, yielding the final version of the system. The last phase will be the compilation of the operating system by the P.O.L. compiler. The "bootstrap" compilers may be any ad hoc compilers, and will be easy and quick to generate.

Alternately, a bootstrap compiler may be written in FORTRAN or COBOL or any other language sufficient to translate the P.O.L. coded compiler into machine code. The quicker and easier method the two will be selected.

The first compiler and operating system for P.O.L. will be developed on the CDC 3600 computer. Since the whole system will be coded in P.O.L., it will be a relatively small effort to compile other P.O.L. systems (designed for other hardware) on the CDC 3600. The specific hardware on which the P.O.L. system is first developed presents no limitations on the "machinability" of P.O.L. in terms of other equipment.

THE OPERATING SYSTEM

The language and the compiler described in the preceding sections clearly limit the type of operating system that is most likely to present an acceptable frame of reference to them. Several versions of the operating system are likely to be developed over the years, some better suited to the principles underlying the language than others. The version to be described here is the contemplated initial implementation.

Past experience indicates that the flexibility of the language and the amount of monitoring done by the system are inversely proportional. The P.O.L. operating system is designed to allow maximum flexibility in the language by performing minimal monitoring.

The structure of the operating system is not like the conventional monitor and many subroutines system, but more nearly resembles one extremely complex conditional expression. There will be a minimum of service routines, each one defined as a single macro-function, to be executed if required.

The system will not be media-bound in any sense: programs may be entered through any of the media (punched cards, paper or magnetic tape, keyboard devices, etc.). The notation of the language is of sufficient syntactic clarity, that no subdivisions (such as card, line of code, etc.) which are not intrinsic to the notation need to be superimposed on it. Similarly, the output may be taken on a variety of media (e.g., punched cards, paper or magnetic tape, line-print, type-written pages, CRT images, etc.). The i/o primitives of the language will be so defined (see above in the section on the language) that service routines involved in the input/output activities of the monitor will be realizable in terms of extremely simple code.

The operating system will perform the queuing of requests from remote consoles, the recognition of the nature of the requests, the storage allocation on all levels of storage usage and on all storage media, the loading of the programs, and any object-time diagnostics that may be called for. Note that only storage usage is monitored by the system; i/o activities and the interrupt system are under the programmer's control.

In a preceding section dealing with the language it was stated that the "program deck" consists of two parts: the description of the computational process, and a function -- part of the operating system, not of the language -- that causes the execution of the program. Further it was stated that the operating system will allow the programmer to slowly "evolve" his program, defining and debugging one macro at a time. To illustrate this process, and to describe the facilities relevant to it, one such effort will be followed from beginning to end.

The user is located at some remote station, which for the sake of this example, will be some CRT-keyboard combination. The user will sign on and wait for acknowledgement from the system. Upon the system's acknowledgement of his identity, he will identify the type of job he contemplates, in this case the beginning of a program generation and debug process. The system will acknowledge this by assigning a specific identification code to the job to be undertaken. At this point, the programmer is ready to begin. He will start by the mnemonic DEFINE and will type in the first macro-function definition. Upon completion signalled by the END mnemonic, he may exercise the following options: he may request the system to read back his definition for visual verification or he may proceed with the compilation or else he may wish to execute. Upon visual verification, the system will return the definition with each function numbered, so that changes and corrections may be effected. When visual verification is complete, a signal from the keyboard will cause compilation and the presentation of diagnostics that may be called for by errors of a syntactic nature. Whether execution is chosen directly, or after a separate run for compilation, it is done in terms of one of the two operating system functions: EXECUTE and LAMBDA. The EXECUTE function is applicable if the defined macro has no arguments and is ready for execution as is. Of course, if the macro calls for any input, that must be readied by the programmer. The LAMBDA function is applicable if the macro just defined has arguments. Using the LAMBDA function, the programmer can define test-values for the arguments that will be used for the given execution. Debugging a particular macro may call for several executions, each with different test-values for the arguments.

During the execution, the programmer will receive object-time diagnostics about errors which are not of a syntactic nature (e.g., reference is made to a function not yet defined, a given variable has gone beyond specified limits, etc.). The programmer may make his corrections and re-execute, until he is satisfied with the macro. He then may place the macro in temporary storage assigned to this particular job by the system and may continue or sign off.

To resume the job at some later time, the user must use the job-specific identification code when signing on. This will make available to him the macros that he has already defined, and he may proceed to use any of his previously defined macros in the definitions of other macros.

This process may be repeated until the complete program is evolved. Any of the macros defined may be entered as a system-macro, which will cause the compiler to incorporate the macro-definition and make it available to any of the users of the system (any system-macro entered from a remote station will be logged in the central facility for book-keeping and documentation purposes). It is in terms of sets of such system-macros that problem-oriented terminologies may be evolved.

When the program is completed and debugged, it may be taken out in any output medium, or may be placed in a special user-file in the system, from which it may be loaded and executed upon request from the user.

Whether the program is called from a user-file or from one of the input media, the execution procedure will be much the same: the operator or the user will request the execution by identifying his job-number, the specific user-file or input medium containing the program, and then the user or the operator may initiate execution by the use of the EXECUTE or the LAMBDA function.

If, during any of the executions, some configurational change in the equipment causes an error, the user or the operator will have the opportunity to enter the appropriate corrections. When a program or a system-macro is no longer useful, it may be removed from the system or the user-file, making the space available for programs of more current interest.

For many eventualities either related to or independent from the program, the programmer may make provisions by the use of the COND quasi-functions. If the programmer choses not to make such provisions, or if no provisions can be made, it will be the job of the operating system to initiate such action as may be appropriate. Thus, functions will be included to handle the emergencies that may occur during the execution of the program. Some of these will only result in the loss of some time, some in the loss of position in the queue, others yet may cause the termination of the program, requiring a complete restart.

Another portion of the operating system is the dynamic storage allocator. The DSA is in complete charge of the storage media and supervises the loading of programs, and their storage usage in execution. Though the programmer has some control over what he places on disks, drums or tapes by issuing WRITE functions, the allocator may place other objects of his program on one of the peripheral media and access them from there upon request or reference. During debugging, if dumps are requested, they will be taken with the aid of the DSA, since a straight storage dump is not likely to be of much use to the programmer without a storage-map and an assembly listing. Accordingly, dumps of structures may be requested by name, and the allocator will provide them.

In general, the implementation of the diagnostics is not an insurmountable task, since the whole system is designed so that the generation of diagnostics will merely be a by-product of normal functioning.

Throughout the development of the language, its compiler and its operating system, an effort has been made to use the simplest, least restricted and therefore most general models. It is believed that the result of this approach is a more powerful system, realized in terms of a much simpler processor. The bulk of code in most compilers and operating systems is devoted to testing for inadmissible cases that are difficult to distinguish from the admissible ones. In P.O.L., most cases are admissible, and idiosyncratic limitations--if desired--must be introduced by the individual programmers.

The Documentation

A variety of reference manuals and texts will be developed for potential users of the P.O.L. system, at various levels of sophistication. One such document will be the annotated source-language listing of the P.O.L. compiler and of the operating system, with specific instructions for effecting desired modifications. In addition, suggestions will be offered for the documentation and circulation of system-macros that may be created later, in the course of using the system.

Some materials will be made available in the form of an introduction to P.O.L., explaining some of the notions and techniques that a programmer must know in order to use P.O.L. to the full extent of its capabilities.

The whole of the language will be formalized in terms of some device like Smullyan's elementary formal systems. The formalization will go beyond the conventional compiler to translate the notation to some variant of a Church or Curry lambda-notation, and will define the properties of the language down to the level of basic axioms. The formalization will consist of an explication of the operands and their properties, the operators (arithmetic and logical, etc.) and their properties, and the primitives, their range and domain and the nature of the mappings obtaining from domain to range. In addition to this, there will be procedures offered for the formalizations of macro-functions.

A syntax of the P.O.L. notation will also be constructed, containing rules that state the constraints on values that objects in certain positions may assume.

Footnote to page 13 and subsequent pages:

The syntactic notation conventions:

1.) bold letter items denote themselves (i. e., are symbols of the language)

2.) lower case items denote a class of items in the language

3.) items enclosed in / . . . / brackets are optional.

If there are several items listed in a vertical column between / . . . / brackets, one of the listed items must be selected.

If only one item is given in the brackets, the item may be present or omitted optionally.

4.) the comma is used as a concatenation symbol, delimiting elements of the syntactic string.

APPENDIX

Sample Problems Coded in P.O.L.

The purpose of this section is to offer some idea of the appearance of the P.O.L. code and the "feel" of the language. The algorithms are not necessarily the best ones for the solutions of the given problems; they were prepared to illustrate some of the P.O.L. functions in context.

Problem 1. Solution of a quadratic equation of the form $ax^2 + bx + c = 0$ for both real and complex numbers.

```
DEFINE  QUADREQ  (A) (B) (C) (REAL1) (REAL2) (COMPLEX1) (COMPLEX2)
                 (TYPE)
   CMNT   A, B, AND C ARE THE INPUTS TO THE MACRO, R1, R2, C1,
AND C2 ARE THE OUTPUTS, AND TYPE IS A SWITCH INDICATING WHETHER
THERE ARE ANY COMPLEX ROOTS OR NOT  CMNT
   LET DISCR ATOM 1WD
   LET X ATOM 1WD
   SET (DISCR)  (B ** 2 - 4 * A * C)
   SET (TYPE)  (ATOM HOL REAL)
   IF (DISCR NE 0)  (IF  (DISCR LT 0)  (SET  (DISCR)  (- DISCR)
                                        SET  (X)  (SQRT DISCR)
                                        SET  (TYPE)  (ATOM HOL CPLX)
                                        SET  (Y)  (2 * A)
                                        SET  (REAL1)  (REAL2) (- B / Y)
                                        SET (COMPLEX1)  (X / Y)
```

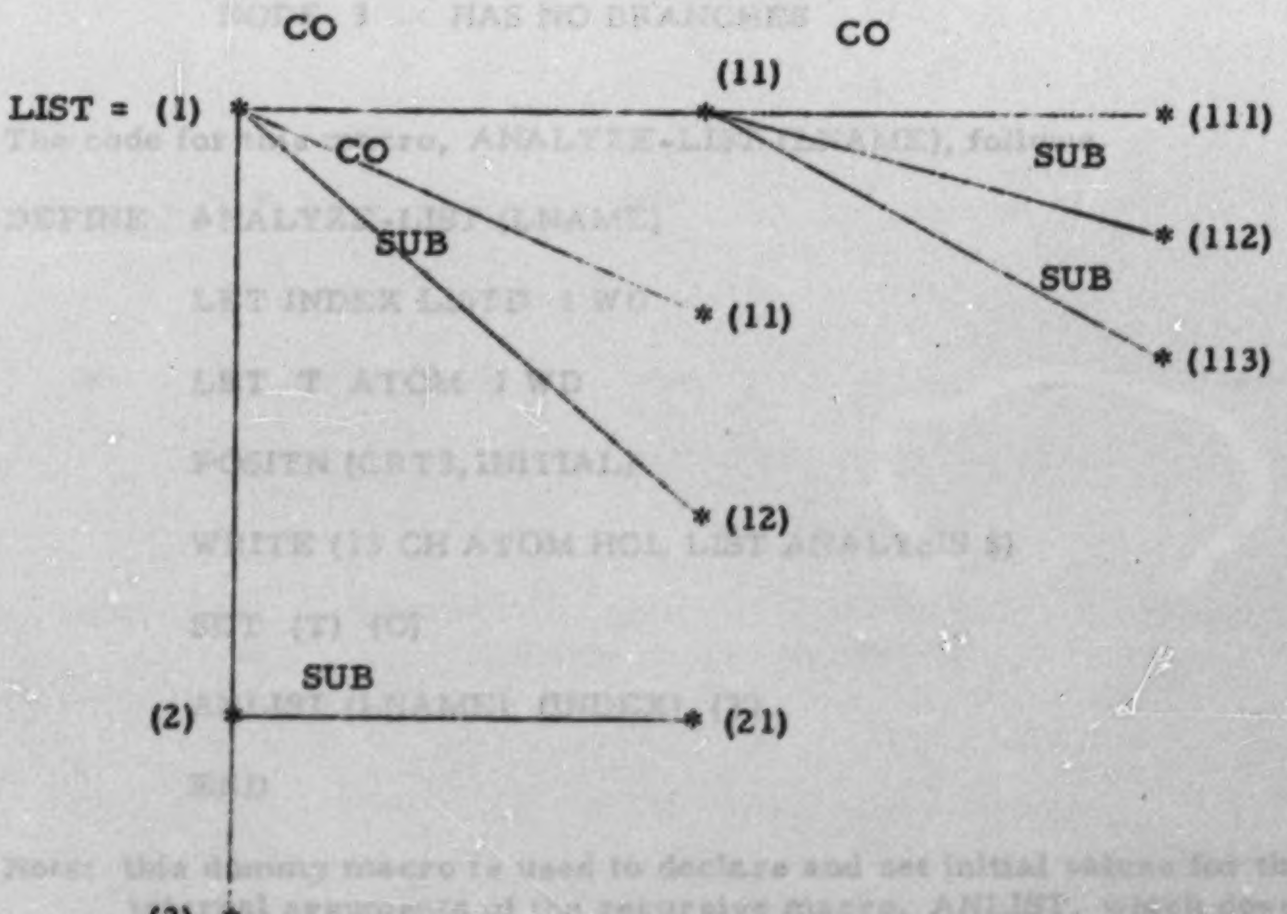
```
                                   SET (COMPLEX2)  (- COMPLEX1)
                                   GOTO  (END) )
   CMNT AT THIS POINT WE HAVE LEFT THE INSIDE IF AND RETURNED
TO THE OUTSIDE IF  CMNT
                                 SET (COMPLEX1)  (COMPLEX2)
                                     (ATOM SPF O)
                                 SET (X)  (SQRT DISCR)
                                 SET (REAL1)  ( (-B + X) / (2 * A) )
                                 SET (REAL2)  ( (-B - X) / (2 * A) )
                                 GOTO (END)
          (T) (SET (COMPLEX1)  (COMPLEX2)  (ATOM SPF O)
              SET  (REAL1)  (REAL2)  (- B / (2 * A) )))
 END
```

coded by S. Kent Brown

NOTE: the indentation is not part of the code and has been included here merely for the sake of clarity.

Problem 2. Analysis of a list-structure that has been generated elsewhere in the program, such that the macro-function below is cognizant only of the name of the stem of the structure and will generate a complete structural description of the structure suppended from that list. The function is defined recursively; the listing it yields does not distinguish coordinate branches from one another (since that would imply an ordering that is not characteristic to these objects).

For the list - structure:



the function ANALYZE-LIST (LIST) would produce the time-sequential listing:

```
"LIST ANALYSIS
    NODE  1     HAS  2  CO BRANCHES
    NODE  11     HAS  1  CO BRANCHES
    NODE  111      HAS NO BRANCHES
NODE 11     HAS 2 SUB BRANCHES

NODE 112      HAS NO BRANCHES

NODE 113      HAS NO BRANCHES

NODE 11     HAS NO BRANCHES

NODE 1    HAS 1 SUB BRANCHES

NODE 12     HAS NO BRANCHES

NODE 2    HAS 1 SUB BRANCHES

NODE 21     HAS NO BRANCHES

NODE 3    HAS NO BRANCHES
```

The code for this macro, ANALYZE-LIST (LNAME), follows.

```
DEFINE   ANALYZE-LIST (LNAME)

         LET INDEX LISTD  1 WD

         LET  T  ATOM  1 WD

         POSITN (CRT3, INITIAL)

         WRITE (13 CH ATOM HOL LIST ANALYSIS $)

         SET  (T)  (O)

         ANLIST (LNAME)  (INDEX)  (T)

         END
```

Note: this dummy macro is used to declare and set initial values for the internal arguments of the recursive macro, ANLIST, which does the actual list analysis.

```
DEFINE   ANLIST (LNAME)  (INDEX)  (T)
         LET P LISTD  1 WD
         LET Q LISTD  1 WD
   CMNT    INITIALIZE    CMNT
         SET  (A)  (INT COUNT LNAME)
         SET  (C)  (1 + INT COUNT INDEX)
         SET  (B)  (ATOM INT 1)
         IF (T EQ 2) (SET (G) (ATOM INT 2)
                     MOVE (G) (INDEX, UNDER)
                     GOTO (SKIP)  )
         SET (H)  (ATOM INT 1)
         MOVE (H) (INDEX, UNDER)
SKIP     SET (J) (ATOM INT 1)
  CMNT    BEGIN SECTION FOR LISTING NODES  CMNT
LOOP     POSITN (CRT3, INITIAL)    CMNT INITIAL CONTAINS THE INITIAL
                                       X, Y  COORD.  CMNT
         WRITE  (8 CH ATOM HOL bbNODEbb $)  (CRT3)
ITER     IF  (J LE C)  (WRITE (3 CH ATOM INT   INDEX(J)   $)  (CRT3)
                      SET  (J)  (J + 1)
                      GOTO  (ITER)  )
         WRITE (6 CH ATOM HOL   bbbbbb  $)  (CRT3)
         CMNT TEST NODE FOR BRANCHES, AND RECURSE ON FINDING
             ANY    CMNT
         IF  (LNAME(O B)  CO)  (MOVE (ACCESS)  (P)  CMNT   IN LNAME(O B),
                                                    THE O FORCES 1-ORI-
                                                    GIN INDEXING  CMNT
                                SET (SAVE)  (T)
                                SET  (T)  (1)
                                SET (E)  (INT COUNT P)
                                WRITE  (5 CH ATOM HOL  HASbb   $) (CRT3)
                                WRITE  (3 CH ATOM INT  E   $)  (CRT3)
                                WRITE  (15 CH ATOM HOL  bbCObBRAN-
                                                  CHESbb   $)  (CRT3)
                                ANLIST  (P) (INDEX) (T)
                                IF  (NOT  (LNAME(O B) SUB)  ) (SET (T)
                                                                  (SAVE)
                                                                GOTO
                                                              (CONTINUE)  )
                                SET  (T)  (2)  )
         IF  (LNAME (O B) SUB)  (MOVE (ACCESS)  (Q)
                                  IF (T NE 2)  (SET (SAVE) (T)
                                              SET (T)  (O)
                                              GOTO  (PASS)  )
                                  POSITN (CRT3) (INITIAL)
                                  WRITE  (8 CH ATOM HOL bbNODEbb  $)
                                                                   (CRT3)
                                  SET  (J) (1)
                       ITER2      IF (J LE C)  (WRITE (3 CH ATOM INT
                                                  INDEX(J)   $)  (CRT3)
                                              SET (J) (J + 1)
                                              GOTO  (ITER2)
                                  WRITE (6 CH ATOM HOL bbbbbb  $) (CRT3)
                          PASS    SET  (F)  (INT COUNT Q)
                                  WRITE (5 CH ATOM HOL HASbb  $) (CRT3)
                                  WRITE (3 CH ATOM INT  F   $) (CRT3)
                                WRITE (14 CH ATOM HOL
                                          bbSUBbBRANCHES  $) (CRT3)
                                ANLIST (Q)  (INDEX)  (T)
                                SET (T) (SAVE)
                                GOTO  (CONTINUE)  )
          WRITE  (15 CH ATOM HOL   HASbNObBRANCHES   $) (CRT3)
          CMNT   SET ITERATION  CMNT
CONTINUE   SET  (B)   (B + 1)
          SET (J)  (1)
          IF (B LE A)  (IF (NOT (T EQ 1)  (SET (INDEX(C)  )  (INDEX(C) + 1)  )
                      GOTO  (LOOP)
          CMNT  CLEAN-UP   CMNT
          MOVE  (INDEX(C)  ) (ERASE LIN)

          END
```

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